# Infinite spin fields in the frame-like formalism

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# Outlook

1. Infinite spin fields in 3d
   - Lagrangian formalism
   - Unfolded formulation
   - Supersymmetry
2. Infinite spin fields in $d = 4$
3. Infinite spin fields in $d \geq 5$

# Poincare group representations

- Massive $p^2 = m^2 \Leftrightarrow$ little group $SO(d-1)$
- Massless $p^2 = 0 \Leftrightarrow$ little group $SO(d-2)$, $T_i$, $i = 1, 2, \dots d-2$
  - finite dimensional $T_i = 0 \Leftrightarrow SO(d-2)$
  - infinite dimensional $T_i \neq 0 \Leftrightarrow$ short little group $SO(d-3)$
- $d = 4$ example
  - massive spin $s$: $0, \pm 1, \pm 2, \dots, \pm s$
  - finite massless: $\pm s$
  - infinite massless: $0, \pm 1, \pm 2, \dots, \pm s, \pm s + 1, \dots$
- Super Poincare group representations: finite massless, massive, infinite massless

# Fermionic case — construction

- Fields set: one-forms $\Psi^{\alpha(2k+1)}$, $0 \le k < \infty$ and zero-form $\psi^\alpha$
- Sum of the kinetic and initial gauge transformations:

$$\frac{1}{i}\mathcal{L}_0 = \sum_{k=0}^{\infty} \frac{(-1)^{k+1}}{2} \Psi_{\alpha(2k+1)} D\Psi^{\alpha(2k+1)} + \frac{1}{2}\psi_\alpha E^\alpha{}_\beta D\psi^\beta$$

$$\delta_0 \Psi^{\alpha(2k+1)} = D\zeta^{\alpha(2k+1)}$$

- Cross terms:

$$\frac{1}{i}\mathcal{L}_1 = \sum_{k=1}^{\infty} (-1)^{k+1} c_k \Psi_{\alpha(2k-1)\beta(2)} e^{\beta(2)} \Psi^{\alpha(2k-1)} + c_0 \Psi_\alpha E^\alpha{}_\beta \psi^\beta$$

- Mass-like terms:

$$\frac{1}{i}\mathcal{L}_2 = \sum_{k=0}^{\infty} (-1)^{k+1} \frac{d_k}{2} \Psi_{\alpha(2k)\beta} e^\beta{}_\gamma \Psi^{\alpha(2k)\gamma} - \frac{m_0}{2} E\psi_\alpha\psi^\alpha$$

# Fermionic case — solution

- General solution:

$$d_k = \frac{m_0}{(2k+3)}, \qquad {c_k}^2 = \frac{(2k+1)}{4(k+1)}{c_0}^2 - \frac{k}{2(2k+1)}{m_0}^2$$

- Three possibilities:
  - ${m_0}^2 > 2{c_0}^2$ In general non-unitary but if $c_{k_0} = 0$ we obtain massive field with spin $s = k_0 + 3/2$
  - Unitary massless infinite spin:

$$ {m_0}^2 = 2{c_0}^2 \quad \Rightarrow \quad {c_k}^2 = \frac{{c_0}^2}{4(k+1)(2k+1)}$$

  - ${m_0}^2 < 2{c_0}^2$ — tachionic case

# Bosonic case

- Fields set: one-forms $\Omega^{\alpha(2k)}$, $\Phi^{\alpha(2k)}$ $1 \le k < \infty$, $A$ and zero-forms $B^{\alpha(2)}$, $\pi^{\alpha(2)}$, $\varphi$
- General pattern:

$$\mathcal{L} = \mathcal{L}_{kinetic} + \mathcal{L}_{cross} + \mathcal{L}_{mass-like}$$

- General solution has two free parameters with the same set of solutions

# Gauge invariant objects

- Gauge invariant two-forms:

$$\mathcal{F}^{\alpha(2k+1)} = D\Psi^{\alpha(2k+1)} + \frac{d_k}{(2k+1)} e^\alpha{}_\beta \Psi^{\alpha(2k)\beta} + c_{k+1} e_{\beta(2)} \Psi^{\alpha(2k+1)\beta(2)} + \frac{c_k}{k(2k+1)} e^{\alpha(2)} \Psi^{\alpha(2k-1)}$$

$$\mathcal{F}^\alpha = D\Psi^\alpha + d_0 e^\alpha{}_\beta \Psi^\beta + c_1 e_{\beta(2)} \Psi^{\alpha\beta(2)} - c_0{}^2 E^\alpha{}_\beta \psi^\beta$$

- Gauge invariant one-form:

$$\mathcal{C}^\alpha = D\psi^\alpha - \Psi^\alpha + d_0 e^\alpha{}_\beta \psi^\beta + c_1 e_{\beta(2)} \psi^{\alpha\beta(2)}, \qquad \delta\psi^{\alpha(3)} = \zeta^{\alpha(3)}$$

- General gauge invariant one-forms ($k \geq 2$):

$$\mathcal{C}^{\alpha(2k+1)} = D\psi^{\alpha(2k+1)} - \Psi^{\alpha(2k+1)} + \frac{d_k}{(2k+1)} e^\alpha{}_\beta \psi^{\alpha(2k)\beta} + c_{k+1} e_{\beta(2)} \psi^{\alpha(2k+1)\beta(2)} + \frac{c_k}{k(2k+1)} e^{\alpha(2)} \psi^{\alpha(2k-1)}$$

$$\delta\psi^{\alpha(2k+1)} = \zeta^{\alpha(2k+1)}$$

# Lagrangian and unfolded equations

- Lagrangian in terms of gauge invariant objects:

$$\mathcal{L} = \sum_{k=0}^{\infty} (-1)^{k+1} \mathcal{C}_{\alpha(2k+1)} \mathcal{F}^{\alpha(2k+1)}$$

- Extra fields decoupling conditions:

$$\frac{\delta \mathcal{L}}{\delta \psi^{\alpha(2k+1)}} = 0, \qquad k \geq 1$$

- Unfolded equations:

$$\mathcal{F}^{\alpha(2k+1)} = 0, \qquad \mathcal{C}^{\alpha(2k+1)} = 0, \qquad k \geq 0$$

# Bosonic case

- For each field (including all extra fields) one can construct corresponding gauge invariant object (two-form or one-form):

$$\begin{array}{cccccc} \Omega^{\alpha(2k)} & \Phi^{\alpha(2k)} & A & \varphi & B^{\alpha(2k)} & \pi^{\alpha(2k)} \\ \mathcal{R}^{\alpha(2k)} & \mathcal{T}^{\alpha(2k)} & \mathcal{A} & \Phi & \mathcal{B}^{\alpha(2k)} & \Pi^{\alpha(2k)} \end{array}$$

- The free Lagrangian can be rewritten in the explicitly gauge invariant form satisfying extra field decoupling conditions:

$$\mathcal{L}_0 = \sum_{k=1}^{\infty} (-1)^{k+1} [\Pi_{\alpha(2k)} \mathcal{R}^{\alpha(2k)} + \mathcal{B}_{\alpha(2k)} \mathcal{T}^{\alpha(2k)}]$$

- Unfolded equations:

$$\mathcal{R}^{\alpha(2k)} = 0, \qquad \mathcal{T}^{\alpha(2k)} = 0 \qquad \dots$$

# Supertransformations

- General pattern is the same as for any supermultiplet

$$
\begin{aligned}
\delta\Phi^{\alpha(2)} &= i\alpha_k \Psi^{\alpha(2k+1)\beta}\zeta_\beta + i\beta_k \Psi^{\alpha(2k-1)}\zeta^\alpha \\
\delta\Psi^{\alpha(2k+1)} &= \gamma_k \Phi^{\alpha(2k+1)\beta}\zeta_\beta + \delta_k \Phi^{\alpha(2k)}\zeta^\alpha
\end{aligned}
$$

- All coefficients are completely determined by the requirement that all gauge invariant objects transform covariantly under supertransformations:

$$
\begin{aligned}
\delta\mathcal{T}^{\alpha(2)} &= i\alpha_k \mathcal{F}^{\alpha(2k+1)\beta}\zeta_\beta + i\beta_k \mathcal{F}^{\alpha(2k-1)}\zeta^\alpha \\
\delta\mathcal{F}^{\alpha(2k+1)} &= \gamma_k \mathcal{T}^{\alpha(2k+1)\beta}\zeta_\beta + \delta_k \mathcal{T}^{\alpha(2k)}\zeta^\alpha
\end{aligned}
$$

- Also supersymmetry requires that the two main parameters for boson and fermion must be equal:

$$
a_0 = c_0 \quad \Leftrightarrow \quad M_B = M_F
$$

# Infinite spin fields in $d = 4$

- As in $d = 3$ we have just one bosonic and one fermionic cases and their Lagrangian formulation can be constructed starting with gauge invariant formulation for massive fields corresponding to completely symmetric (spin)tensors in the limit where mass goes to zero and spin goes to infinity — Metsaev 2016, 2017
- Frame-like formalism can be straightforwardly obtained by the similar procedure from Zinoviev 2008
- Unfolded formalism
  - bosons — Ponomarev, Vasiliev 2010

$$\Phi_\mu{}^{a(k),b(l)}, \quad S^{a(k),b(l)} \quad 0 \leq k \leq s-1, \quad 0 \leq l \leq k$$

$$W^{a(k),b(l)} \quad k \geq s, \quad 0 \leq l \leq s-1$$

  - fermions — ???

# Supersymmetry

- Gauge invariant formulation for the massive $N = 1$ supermultiplets with arbitrary (super)spins exists – Zinoviev 2007

$$\begin{pmatrix} & s+\frac{1}{2} & \\ s & & s' \\ & s-\frac{1}{2} & \end{pmatrix} \qquad \begin{pmatrix} & s+1 & \\ s+\frac{1}{2} & & s+\frac{1}{2} \\ & s & \end{pmatrix}$$

- Spectrum of massless supermultiplets

$$\begin{pmatrix} & \Phi_{s+\frac{1}{2}} & \\ A_s & & B_s \\ & \Psi_{s-\frac{1}{2}} & \end{pmatrix} \Rightarrow \sum_{k=1}^{s} \begin{pmatrix} & \Phi_{k+\frac{1}{2}} & \\ A_k & & B_k \\ & \Psi_{k-\frac{1}{2}} & \end{pmatrix} \oplus \begin{pmatrix} \Phi_{\frac{1}{2}} \\ z \end{pmatrix}$$

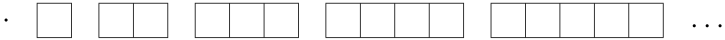
$$\begin{pmatrix} & A_{s+1} & \\ \Phi_{s+\frac{1}{2}} & & \Psi_{s+\frac{1}{2}} \\ & B_s & \end{pmatrix} \Rightarrow \begin{pmatrix} A_{s+1} \\ \Psi_{s+\frac{1}{2}} \end{pmatrix} \oplus \sum_{k=1}^{s} \begin{pmatrix} & \Phi_{k+\frac{1}{2}} & \\ A_k & & B_k \\ & \Psi_{k-\frac{1}{2}} & \end{pmatrix} \oplus \begin{pmatrix} \Phi_{\frac{1}{2}} \\ z \end{pmatrix}$$

# Infinite spin fields in $d \geq 5$

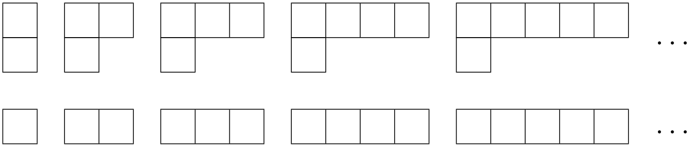
- In $d \geq 5$ we have an infinite number of such infinite spin representations labeled by the representations of the so called short Little group $SO(d-3)$
- For example in $d = 5, 6$ they are labeled by one parameter $l = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots$
- Explicit realization can be obtained starting from the gauge invariant description of massive mixed symmetry fields corresponding to the Young tableau with two rows $Y(k, l)$, $k > l$ (Zinoviev 2008, 2009) and taking the limit where mass goes to zero, $k- > \infty$ while $l$ being fixed

# Examples of spectrum

- Completely symmetric case $l = 0$

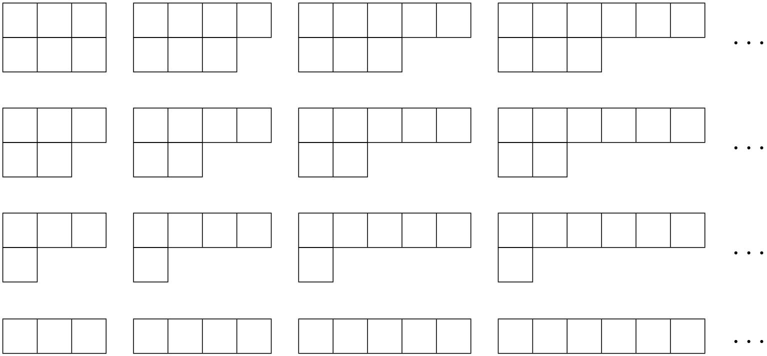


- First non-trivial case $l = 1$



# Examples (cont.)

- Case $l = 3$



# Conclusion

- For the description of such infinite spin massless fields we can use the effective formalism developed for the gauge invariant description of massive higher spin fields (both in metric-like and frame-like versions).
- As a result we can try to investigate possible interactions for such fields again using the same approaches as for the massive ones.
- Similarly to the massive fields such massless infinite spin fields have important dimension-full parameter so it may happens that it is not nesesary to move the flat Minkowski space to *AdS*.