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Restriction on the charge of Q-balls due to radiative corrections

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on arXiv:1612.00700 in collaboration with A. Kovtun

Inhomogeneous field configuration

motivation:

- theory with unstable condensate (gravity, for example), Bose star/Axion star,

  (R. Ruffni and S. Bonazzola, PRD 1969, E. W. Kolb and I. I. Tkachev, PRL 1993)

- DM candidate (soliton), to avoid restrictions on WIMP

  (SUSY, A. Kusenko and M. Shaposhnikov, K. Enqvist and J. McDonald, PLB 1998)

- baryogenesis, cogenesis, PBH production.

  (S. Kasuya and M. Kawasaki, PRD 2014, E. Cotner and A. Kusenko, PRD 2016)

- Fuzzy DM

  (W. Hu, R. Barkana, and A. Gruzinov, PRL 2000) (see also review L. Hui et al., PRD 2017)

main issue:

- stability (especially in the presence of other fields)

Stability of lump   *Charge* or Topology

Static solutions in theories with $V \geq 0 \rightarrow$ problem with Derrick theorem

(nonlinear kinetic term, gauge fields)

For pure scalar field theory scaling arguments restrict number of space-time dimensions $D < 3$

unstable condensate – even in theory with selfinteraction (V. E. Zakharov, JETP 1968)

Let us turn off gravity.

Stationary (but not static!) solution for $U(1)$-invariant scalar field theory:

$$\Phi = e^{i\omega t} f(r)$$

in ordinary $(3+1)$ space-time

only r dependence in f — spherical symmetry.

Energy and Charge are indeed static!

Charge (not electric!) → global $U(1)$ symmetry,

G. Rosen, J. Math. Phys. **9** (1968) 996

or Q-balls, S.R. Coleman, Nucl. Phys. B **262** (1985) 263 [Erratum **269** (1986) 744]

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi - V(\Phi^* \Phi)$$

Single complex scalar field – Eq. of motion can be studied by method of classical mechanics (overshoot-undershoot method, where r corresponds to time)

thin-wall, like a snowball

Eq. is nonlinear, how to check (numerical) result?

$$\boxed{\mathrm{d}E/\mathrm{d}Q = \omega}$$

here

$$E = \int d^3x (\partial_0 \phi^* \partial_0 \phi + \partial_i \phi^* \partial_i \phi + V),$$

$$Q = \mathrm{i} \int d^3x (\partial_0 \phi^* \phi - \phi^* \partial_0 \phi)$$

generalisation with additional real (massive) mediator scalar field $\rightarrow$

NONTOPOLOGICAL SOLITONS

$$\mathcal{L} = \partial_\mu \Phi^* \partial^\mu \Phi + \frac{1}{2}\partial_\mu \Psi \partial^\mu \Psi - V(\Psi) - \lambda(\Phi^*\Phi)\Psi^2$$

R.Friedberg, T.D. Lee, A.Sirlin, PRD **13**(1976) 2739

Time-dependent background: how to investigate stability?

D.L.T. Anderson, G.H. Derrick, J. Math. Phys. **11** (1970) 1336

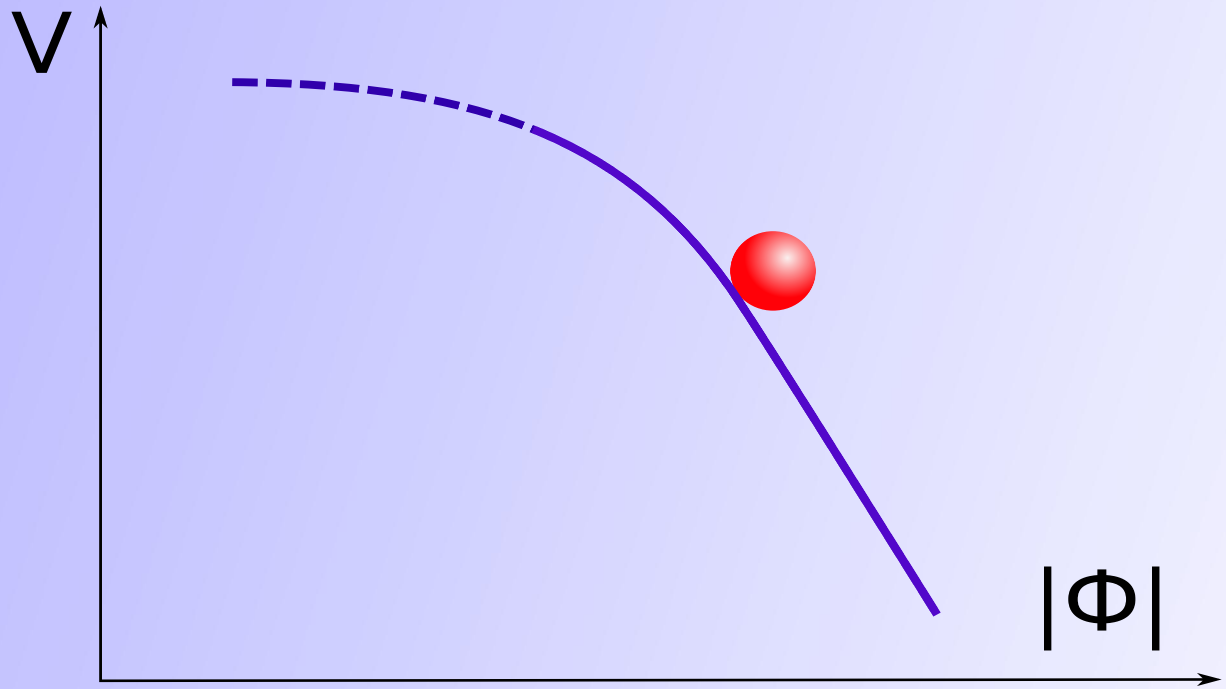
$$\delta\Phi = e^{i\omega t + |\gamma| t}(\psi_1(r) + \psi_2^*(r))$$

$$\begin{pmatrix} \hat{O}_{11} & \hat{O}_{12} \\ \hat{O}_{21} & \hat{O}_{22} \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} = \begin{pmatrix} (\omega-\gamma)^2 \psi_1 \\ (\omega+\gamma)^2 \psi_2 \end{pmatrix}$$

Q-criterion of stability: $\frac{dQ}{d\omega} < 0$

N.G. Vakhitov, A.A. Kolokolov, Radiophys. Quantum Electron. **16** (1973) 783 -NSE

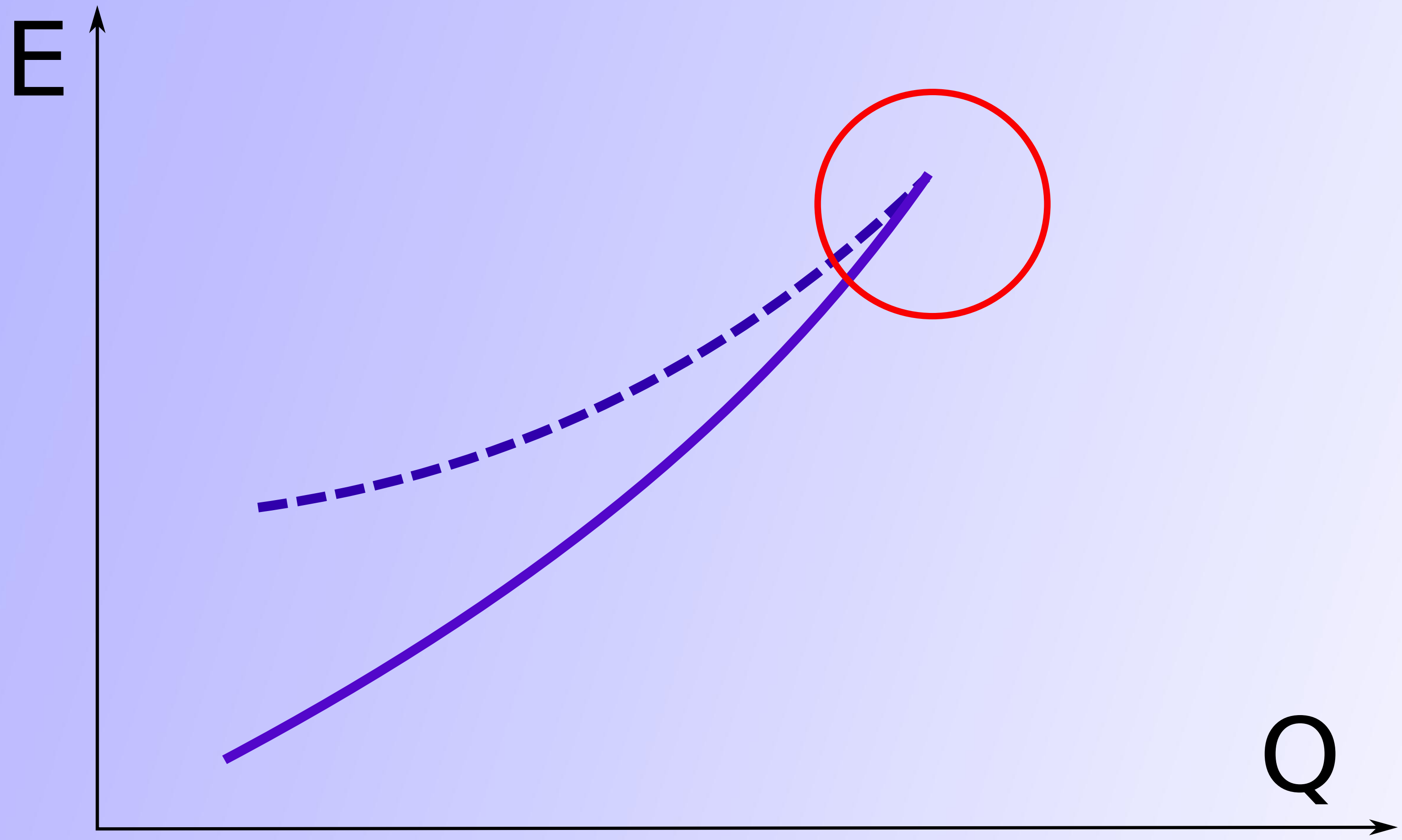
R.Friedberg, T.D. Lee, A.Sirlin, PRD **13**(1976) 2739



For configurations with large amplitude there is a simple interpretation.

Large amplitude $\rightarrow$ large charge (up to 10^{30} for -ph!)

origin of maximal charge Q_{max}



Another origin of restrictions → gauge fields → Coulomb repulsion

(K. M. Lee, J. A. Stein-Schabes, R. Watkins and L. M. Widrow, PRD 1988)

Interaction with charged two-component left-handed spinor χ can be introduced by

$$\mathcal{L} = \chi^\dagger \bar{\sigma}^\mu \partial_\mu \chi - \frac{i\,g}{2}\left(\Phi\,\chi^\dagger \sigma^2 \chi^* - h.c.\right)$$

here $\bar{\sigma}^\mu = (1, -\vec{\sigma})$ and g is small coupling constant

Interesting consequence: Evaporation of Q-balls

(A. G. Cohen, S. R. Coleman, H. Georgi and A. Manohar, Nucl. Phys. B 1986)

which can be treated in thin-wall approximation or numerically

(T. Multamaki and I. Vilja, Nucl. Phys. B 2000)

"This decay process takes place only on the surface of the object, not in the interior. Thus the Q -balls evaporate away."

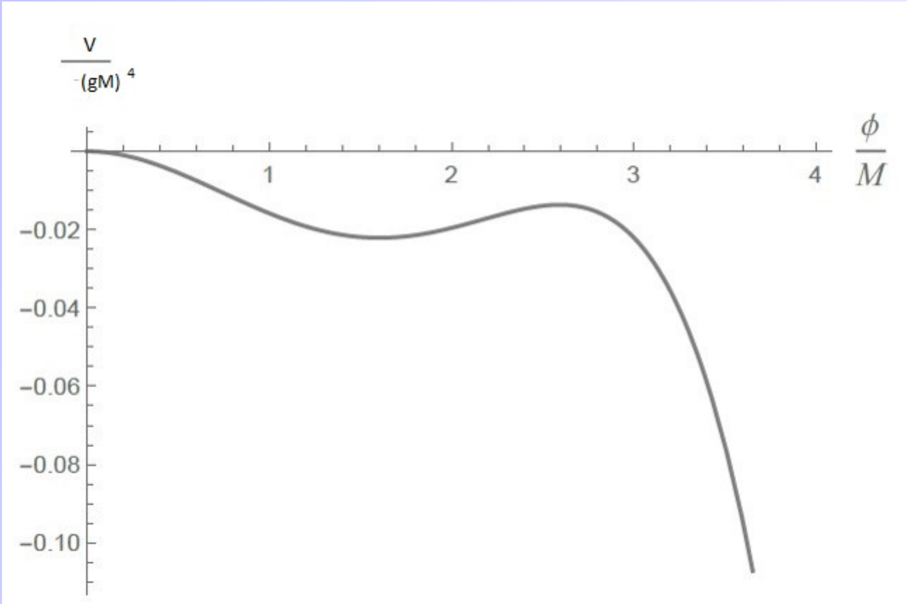
It is not catastrophic process! Without loud clap... In contrast with experience from childhood.

Large charges → large amplitudes... What about Coleman-Weinberg mechanism?

If we consider renormalization conditions

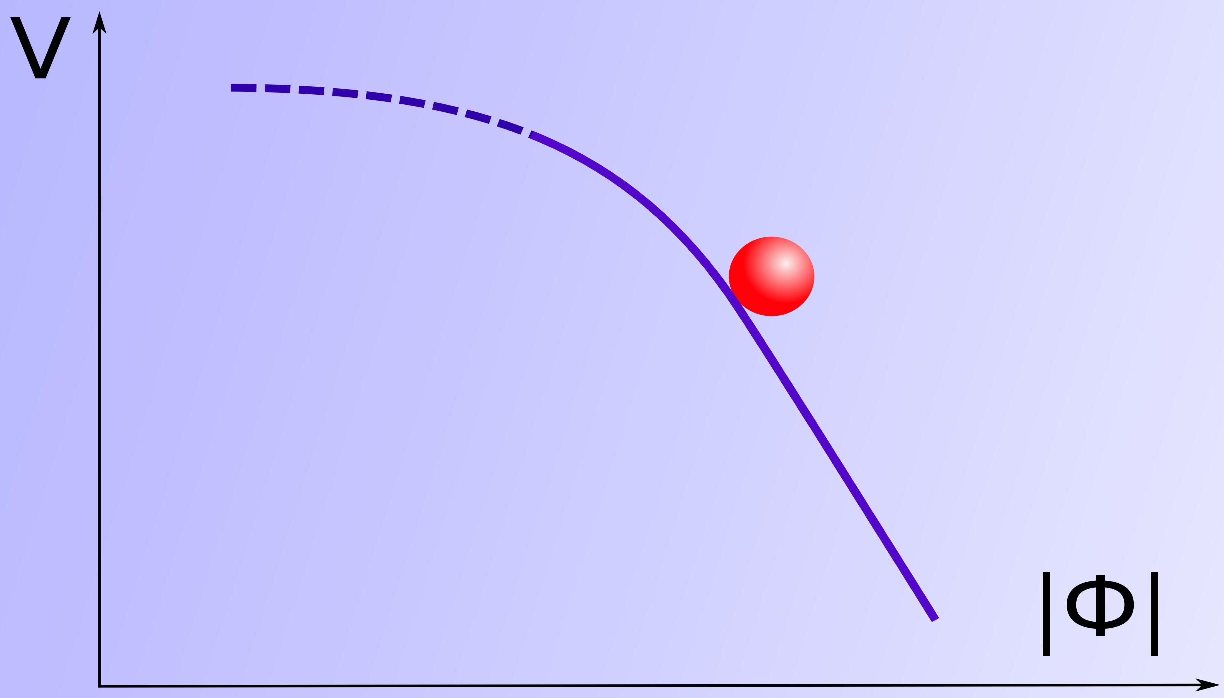
$$\left.\frac{d^2V_{(1loop)}}{d\phi\, d\phi^*}\right|_{\phi=M} = 0, \left.\frac{d^4V_{(1loop)}}{d\phi^2\, d\phi^{*2}}\right|_{\phi=M} = 0$$

correction looks like:



Thus, vacuum is still stable, but for large values of $|\Phi|$ second derivative is negative!

But we have discussed very similar case!



If one take initially flat scalar potential, then Coleman-Weinberg effect will be very essential for configurations with large fields... But it was original potential for Rosen's paper (with analytic solution) and SUSY motivated works!

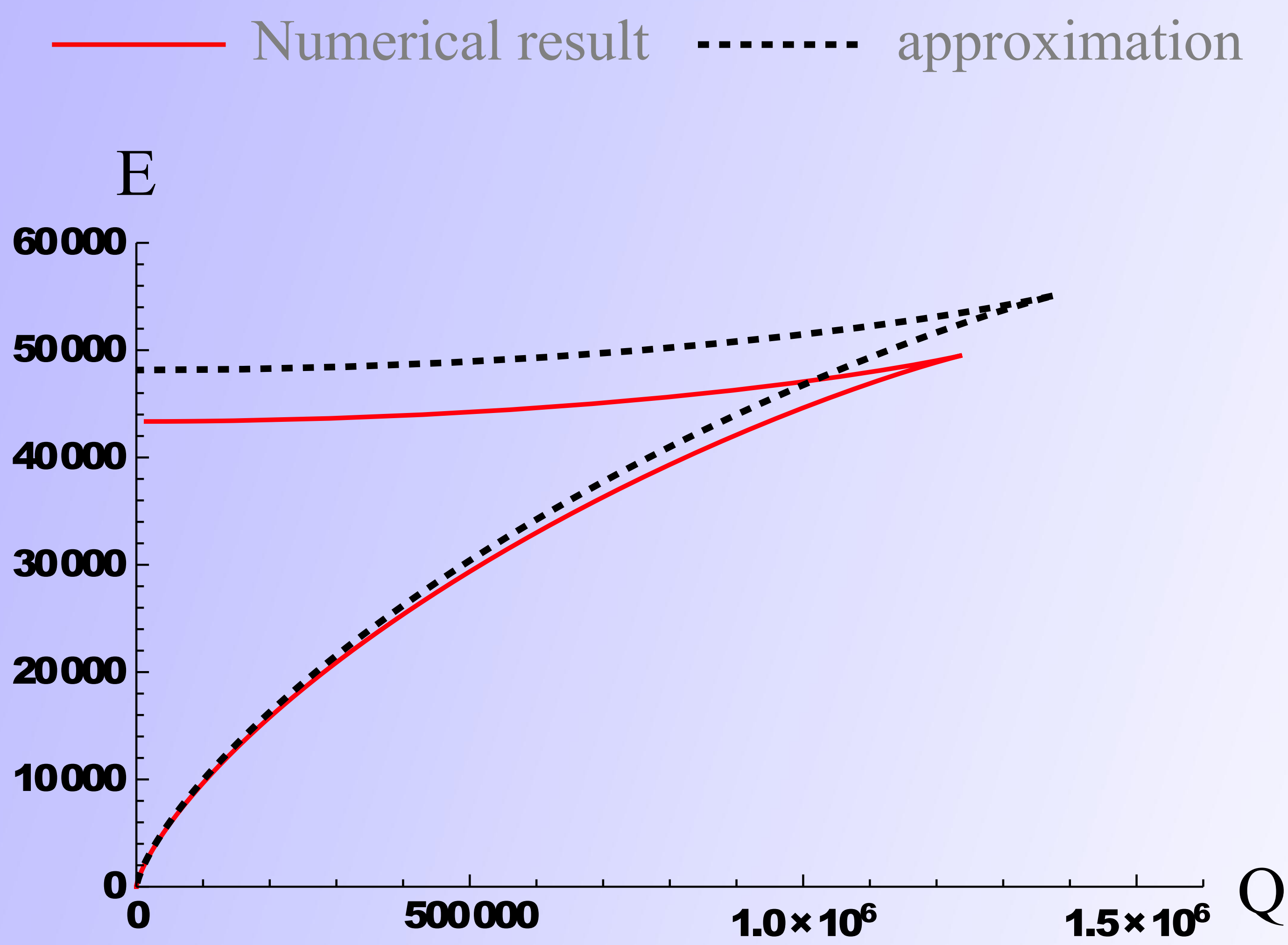
Moreover, for stable configurations

$$\Phi \sim \mathrm{e}^{\mathrm{i}\omega t}, \qquad \frac{\omega}{\sqrt{V''(0)}} \to 0.$$

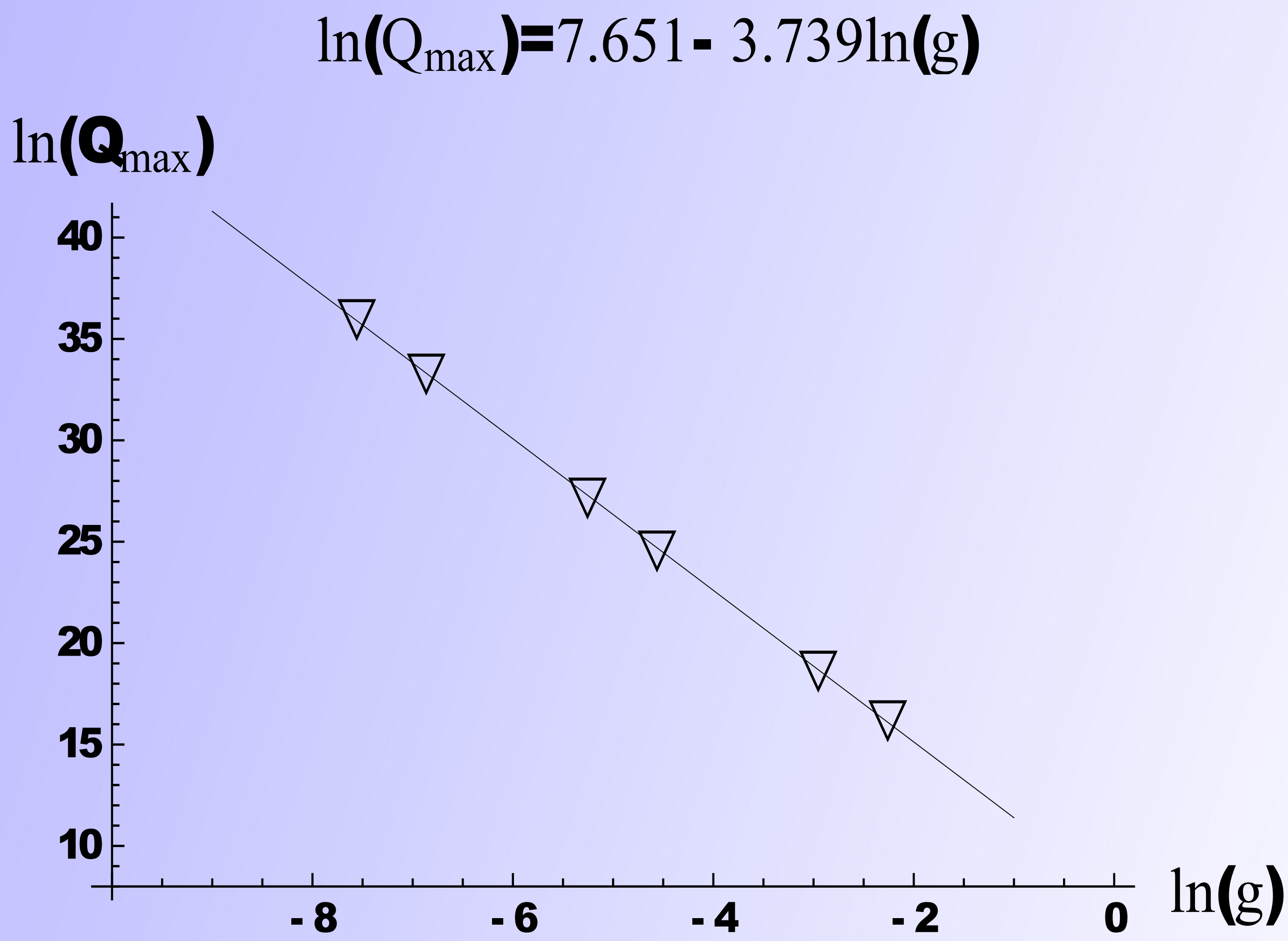
Also, ω determines gradient and one can consider effective potential instead of full effective action.

Potential with one-loop correction does not admit analytic solution. For large charges one should be careful during numerical calculations... We used integral condition $\frac{dE}{dQ} = \omega$ for cross-check and simple model with piecewise potential (E.N and M. Smolyakov, JHEP 2014). In the last case there is a possibility to find dependence $Q_{max}(g)$. Our observations for the critical solution are:

$$\omega_c \sim g \qquad |\Phi(0)| \sim \frac{1}{g}.$$



For integral values approximation works well.



And one can use $Q_{max}(g) \sim \frac{1}{g^4}$ for the model with flat potential.

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Conclusions

- For configurations with large field amplitude Coleman-Weinberg mechanism can play crucial role for stability. It should be noted that homogeneous vacuum $\Phi = 0$ remains classically stable.
- Evaporation of Q-balls should be considered more carefully, because the classical instability evolves exponentially.
- In a toy model one can estimate $Q_{max} \sim \frac{1}{g^4}$.
- Possible ways to avoid restriction: additional bosons.

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THANK YOU!