$c \to \infty$

Colored Gravity in the Non-relativistic Limits

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Outline

- Colored gravity 
- Non-relativistic limits $c \to \infty$
- Non-relativistic colored gravity  $c \to \infty$

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Colored gravity

- Maxwell theory: single massless spin-1 field

  Einstein's gravity: single massless spin-2 field

- Yang-Mills theory: multiple massless spin-1 fields
  + internal symmetry

  Colored gravity: multiple massless spin-2 fields
  + internal symmetry

- Colored decoration:
  QCD is Yang-Mills with color gauge group SU(3)
  Colored gravity involves color gauge group SU(N)

Colored gravity

- No-go for interacting massless spin-2 fields
  Boulanger-Damour-Gualtieri-Henneaux, 2001

- Assume the gauge symmetry is a tensor product $\mathfrak{g}_i \otimes \mathfrak{g}_c$

$$[\, M_X \otimes \boldsymbol{T}_I, M_Y \otimes \boldsymbol{T}_J \,] = \frac{1}{2} [\, M_X, M_Y \,] \otimes \{\, \boldsymbol{T}_I, \boldsymbol{T}_J \,\} + \frac{1}{2} \{\, M_X, M_Y \,\} \otimes [\, \boldsymbol{T}_I, \boldsymbol{T}_J \,]$$

not defined for Lie algebra

- Let us consider associative algebras $g_1 g_2 \sim g_3$
  - $\mathfrak{g}_c$ associative **color** algebra: $\mathfrak{u}(N)$
  - $\mathfrak{g}_i$ associative algebra containing **isometry**
- Evade the no-go by adding **spin-1** fields

Colored gravity algebra

- In 3d, the AdS algebra so(2,2) is isomorphic to

$$sl(2,R) \oplus sl(2,R)$$

- The AdS algebra can be extended to an associative algebra $gl(2,R) \oplus gl(2,R)$

  by adding two identities $sl(2,R) \oplus \mathrm{id} = gl(2,R)$

- It corresponds to adding two u(1) vector fields
  This is strikingly analogous to double central extensions in the non-relativistic gravities

Chern-Simons formulation

- CS action

$$S[\mathcal{A}] = \frac{\kappa}{4\pi} \int \mathrm{Tr}\Big(\mathcal{A} \wedge d\mathcal{A} + \frac{2}{3}\mathcal{A} \wedge \mathcal{A} \wedge \mathcal{A}\Big)$$

- Gauge algebra

$$(gl_2 \oplus gl_2) \otimes u(N)$$

The 1-form gauge field takes value in the gauge algebra. Schematically, we have $\mathcal{A} = A^a g_a$

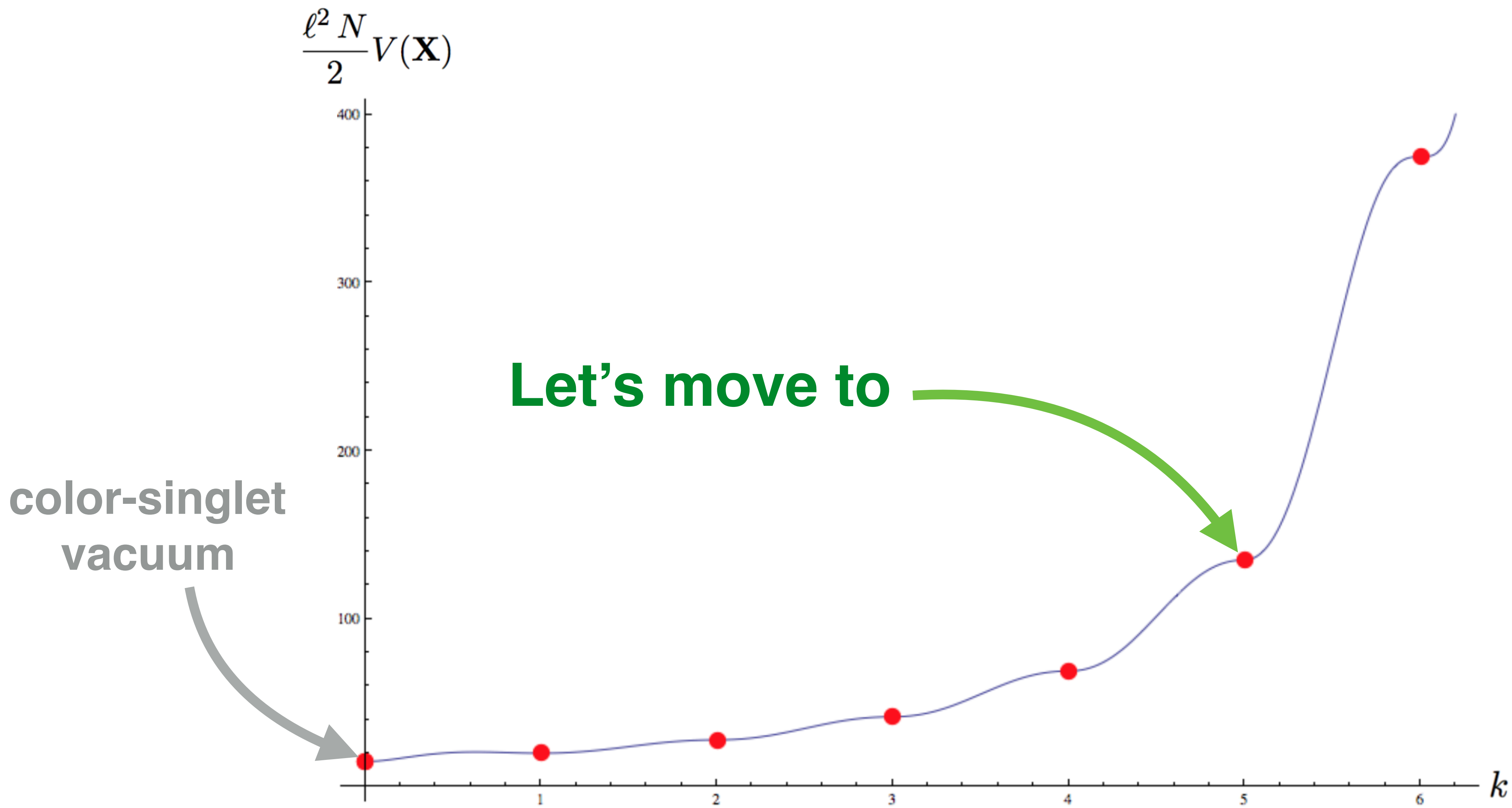
- We can solve the torsionless conditions to obtain the second order formulation

$$D_\mu \boldsymbol{\varphi}_{\nu\rho} = \nabla_\mu \boldsymbol{\varphi}_{\nu\rho} + [\boldsymbol{A}_\mu, \boldsymbol{\varphi}_{\nu\rho}]$$

$$S = S_{\mathrm{CS}} + \frac{1}{16\pi G} \int \mathrm{d}^3x \sqrt{|g|} \left[R - V(\boldsymbol{\varphi}, \tilde{\boldsymbol{\varphi}}) + \frac{2\sqrt{\sigma}}{N\,\ell} \epsilon^{\mu\nu\rho}\, \mathrm{Tr}\left(\boldsymbol{\varphi}_\mu{}^\lambda D_\nu \boldsymbol{\varphi}_{\rho\lambda} - \tilde{\boldsymbol{\varphi}}_\mu{}^\lambda D_\nu \tilde{\boldsymbol{\varphi}}_{\rho\lambda} \right) \right]$$

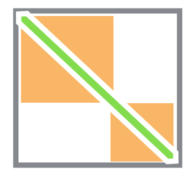
Gwak, Joung, Mkrtchyan, Rey, 2015

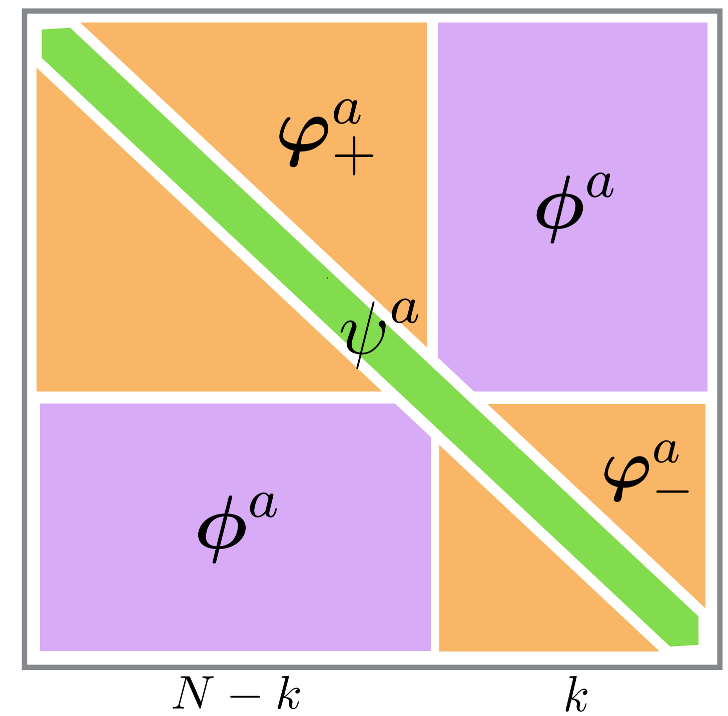
Rainbow valley

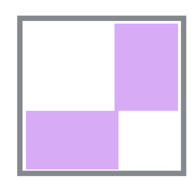


Symmetry Breaking $SU(N-k) \times SU(k) \times U(1)$

Symmetry breaking

- Diagonal part: 
  - adjoint in $SU(N-k) \times SU(k) \times U(1)$
  - remain **massless** spin-two fields



- Symmetry-breaking part: 
  - bi-fundamental
  - combines with (or after eating) the spin-1 fields describe **partially-massless** spin-two fields

- **Higgs**-like mechanism

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Why non-relativistic gravity?

- Gauge-gravity duality (AdS/CMT)
- Condensed matter physics (FQHE)
- Horava-Lifshitz gravity
- Non-relativistic strings & branes

Newton-Cartan gravity

- Newtonian potential
  $\longrightarrow$ frame fields, spin connections + a vector field

$$e_\mu{}^0,\ e_\mu{}^i,\ \omega_\mu{}^{0i},\ \omega_\mu{}^{ij}, \qquad m_\mu$$

- Gauge theory of Bargmann algebra (Andringa, Panda, de Roo, Bergshoeff, 2011) where Galilei algebra is centrally extended by a **mass** generator

  [Galilean boosts, spatial translations] ~ Z

- One-form gauge field $\quad A = e^a P_a + \omega^{ab} M_{ab} + mZ$

- Consider a non-relativistic particle. The Lagrangian is not invariant under Galilean boosts. The central extension is induced by the modified Noether charges.

3d Gravity

- Gravity in 2+1 dimensions is special

- Ricci tensor and Riemann tensor have the same number of independent components (#=6)

  Vacuum solutions have constant curvature
  No propagating degrees of freedom
  No gravitational waves

- Geodesic equations in the Newtonian limit does not involve the Newtonian potential

  In 3d, massive particles do not feel Newtonian gravity

Second central extension

- In 3d, there exists another central extension of the Galilei group
- It is subtly interpreted as a **spin** generator (Jackiw, Nair 2000)
- Galilei + Mass = Bargmann
  Bargmann + Spin = extended Bargmann
- Extended Bargmann algebra has a trace
  (a non-degenerate, invariant bilinear form)
- Extended Bargmann gravity has an action principle
  in terms of a Chern-Simons action
  Bergshoeff-Rosseel, Hartong-Lei-Obers, 2016

Newton-Hooke algebra

- Inonu-Wigner contractions (Bergshoeff-Rosseel 2016)

Poincare	$\rightarrow$	Galilei
Poincare + u(1)	$\rightarrow$	Bargmann
Poincare + u(1) + u(1)	$\rightarrow$	extended Bargmann

- Contraction limit: infinite speed of light $c \to \infty$

- Turn on cosmological constant
  dS/ AdS $\xrightarrow[c \to \infty]{}$ Newton-Hooke (NH) algebra

- 2+1 d: NH+Mass+Spin = extended Newton-Hooke
  action principle in the Chern-Simons formulation

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Double extensions

- In 3d colored gravity, the isometry algebra is extended to an associative algebra by adding **two u(1)** generators.

  A no-go is evaded and an interacting theory of multiple spin-2 fields is constructed.

- In 3d non-relativistic gravities, **two u(1)** generators are added to the relativistic algebra before taking the non-relativistic limits.

  The resulting non-relativistic algebras have two central extensions (mass & spin) and have well-defined traces (CS action).

- We expect to find the colored decorated versions of Extended Bargmann Gravity and Extended Newton-Hooke Gravity

Consistency conditions of group contractions

- In general, we can assign different "**degrees**" to the contracted generators when $\lambda \to \infty$

$$g = i_0 + i_1 + i_2 + \dots, \qquad j_k = \frac{1}{\lambda^k} i_k$$

- Then we have some consistency conditions

$$[j_{k_1}, j_{k_2}] = \frac{1}{\lambda^{k_1+k_2}}[i_{k_1}, i_{k_2}] = j_{k_1+k_2}$$

- In particular, the **degree-0** generators should form a subalgebra

$$[i_0, i_0] \sim i_0$$

In EB & ENH, the degree-0 generators are the time translation and the spatial rotation generators. (abelian)

Non-relativistic limits of colored gravity gauge algebra

- Gauge algebra

$$(gl_2 \oplus gl_2) \otimes u(N) = (gl_2 \oplus gl_2) \otimes I \oplus (gl_2 \oplus gl_2) \otimes su(N)$$

- Color-singlet coincides with extended Newton-Hooke

$$gl_2 \oplus gl_2 \sim so(2,2) \oplus u(1) \oplus u(1)$$

- The possible non-relativistic limits are classified by additional degree-0 generators from the non-singlet sector
  0) no more generators
  1) 1 more set of generators
  2) 2 more sets of generators: type A, type B

Non-relativistic limits of colored gravity gauge algebra

- Color-singlet (extended Newton-Hooke)

H = non-relativistic time translation = relativistic time translation + **u(1)**
Mass = (relativistic time translation - **u(1)**) / c^2

J = non-relativistic spatial rotation = relativistic spatial rotation + **u(1)**
Spin = (relativistic spatial rotation - **u(1)**) / c^2

- Degree-0 subalgebra

Case 0): H, J
Case 1): H, J, **su(N)**
Case 2) Type A: H, J, **su(N)**, relativistic time translation $\otimes$ **su(N)**
Type B: H, J, **su(N)**, relativistic spatial rotation $\otimes$ **su(N)**

Summary and outlook

- 3d Colored gravity

  Evade the no-go for interacting massless spin-2 field
  by extensions to associative algebras:
  so(2,2) + u(1) + u(1) = gl(2) + gl(2)

- Classify the non-relativistic limits using consistency conditions of Inonu-Wigner contractions

  The degree-0 generators form a subalgebra

- Higgs mechanism in the non-relativistic limits ? (PM?)

Thank you!