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Average value of the cosmic ray injection exponent at Galactic sources

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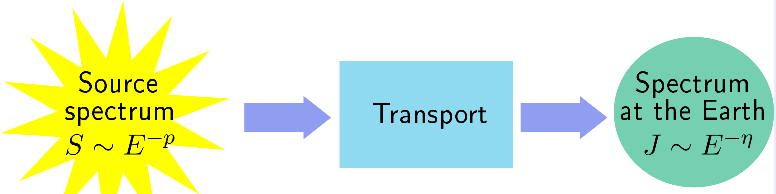
Problem

The spectra of cosmic rays measured at the Earth are different from their source spectra.



$$\eta = p + (?)$$

$$\Downarrow$$

$$p = \eta - (?) \quad \text{or} \quad p = \eta + (?)$$

A key to understanding this difference, being crucial for solving the problem of cosmic-ray origin, is the determination of how cosmic-ray (CR) particles propagate through the turbulent interstellar medium (ISM).

Ginzburg-Syrovatsky's normal diffusion model

If the medium is a quasi-homogeneous the propagation process can be described by a normal diffusion model.

Normal diffusion equation

$$\frac{\partial N(\vec{r},t,E)}{\partial t} = D(E)\Delta N(\vec{r},t,E) + S(\vec{r},t,E). \qquad (1)$$

- $N(\vec{r},t,E)$ is the density of particles with energy E at location $\vec{r}$ and time t;
- $D(E)$ is the diffusion coefficient; $D(E) = D_0 E^\delta$;
- $S(\vec{r},t,E)$ is the distribution density of a galactic sources.

Ginzburg-Syrovatsky's normal diffusion model

Spectrum exponent at the Earth: Steady-state approximation

$$\frac{\partial N}{\partial t} \to 0$$

$$\Downarrow$$

$$N(\vec{r}, E) \sim E^{-\eta} \quad \Rightarrow \quad \eta = p + \delta$$

$$\Downarrow$$

$$p = \eta - \delta$$

$$\delta = 0.3 \quad \Rightarrow \quad \eta = 2.7, \quad p = 2.4$$

$$\delta = 0.7 \quad \Rightarrow \quad \eta = 2.7, \quad p = 2.0$$

However, recent results show that homogeneous diffusion models failed to reproduce observations (see, for example, G. Jóhannesson et al. // ApJ, 2016, **824**).
Multiscale structures in the Galaxy, found during the last few decades, may be taken as a support to this conclusion.
Theory and observations show that the ISM is inhomogeneous (fractal-like) on the hundreds of parsecs scales [1–6]. Stars formation regions also demonstrate fractal features with spatial scales up to about a kpc.

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THE ORIGIN OF COSMIC RAYS

BY

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by the authors in 1964

Motion of cosmic rays in the interstellar medium

What has been said need not be taken to mean that "tangling" of the lines of force guarantees the applicability of the diffusion approximation for calculating the spatial distribution of the particles. That this is not so is clear from the model discussed by Gemantsev (IZV. vyssh. ucheb. zav. Radiofiz., 1962; Astr. zh., 1962; Soviet Astronomy-AJ, 1963). Let us assume that motion occurs only along randomly tangled force tubes but that there is also scattering of the particles in the tubes themselves. For the latter reason we shall consider that the motion along the tube has the nature of one-dimensional diffusion with a diffusion coefficient D'. Then in a time t the particle moves an average distance of $L' = \sqrt{2D't}$ along the tube, i.e., will move at an average effective velocity $v' = L'/t = \sqrt{2D'/t}$. The general shift of the particle in space (as the result of diffusion along the tube and the tube's change in direction) is

$$L = \sqrt{(D't)^{1/2} l/\pi^{1/2}} \propto \sqrt{\frac{2}{3} l v' t} \propto t^{1/4}.$$

instead of that which occurs during ordinary diffusion according to the law $L \propto t^{1/2}$.

From this example (and moreover from other considerations) it is clear that tangling of the force tubes still does not guarantee the validity of the diffusion law $L = t^{1/2}$. We consider, however, that the actual circumstances and arguments provide good reason for using the diffusion model in the Galaxy and not, let us say, Getmantsev's model.

V.L. Ginzburg, S.I. Syrovatskii, P. 175.

Goal

The main goal

The main goal of the report is to retrieve the cosmic ray injection spectrum at the galactic sources taking into account the inhomogeneity of the turbulent interstellar medium.
The spectrum observed at Earth and two different transport models, based on anomalous and normal diffusion equations are used.

Part 1
Anomalous diffusion model

Anomalous diffusion (AD)

A possible way to generalize normal diffusion model is to replace the assumption about statistical homogeneity of inhomogeneities distribution by their fractal distribution. An important consequence of this generalization are:

Lévy flights: The power-law distribution of free paths r in such a medium

$$p(\vec{r}, E) \propto A(E, \alpha) r^{-\alpha-1}, r \to \infty, 0 < \alpha < 2.$$

Lévy trap: The probability density function $q(t, E)$ of time t, during which a particle is trapped in the inhomogeneity, also has a power-law behavior

$$q(t, E) \propto B(E, \beta) t^{-\beta-1}, t \to \infty, \beta < 1.$$

A generalization of the homogeneous normal diffusion model to the case of inhomogeneous (fractal-like) ISM, has been made for the first time in our papers [7,8]. Later, it was shown [9–14] that an anomalous cosmic ray diffusion model, developed by the authors, allows to describe the main features of nuclei, electron and positron spectra observed in the Solar system. Particularly, in the anomalous diffusion model the key feature of the all particle energy spectrum — the knee at $3 \cdot 10^{15}$ eV — appears naturally without additional assumptions.

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Anomalous diffusion equation

Without energy losses ($\alpha \in (0, 2]$, $\beta \in (0, 1]$)

$$\frac{\partial N}{\partial t} = -D(E, \alpha, \beta)\mathrm{D}_{0+}^{1-\beta}(-\Delta)^{\alpha/2}N(\vec{r}, t, E) + S(\vec{r}, t, E), \qquad (2)$$

- $(-\Delta)^{\alpha/2}$ is the fractional Laplacian (Riesz operator);
- D_{0+}^{β} is the Riemann-Liouville fractional derivative;
- $D(E, \alpha, \beta) \sim A(E, \alpha)/B(E, \beta) = D_0(\alpha, \beta)E^{\delta}$ is the anomalous diffusion coefficient.

Riesz operator

$$\int\limits_{\mathrm{R}^m} \mathrm{e}^{ikx}(-\Delta)^{\alpha/2}f(x)\,dx = |k|^{\alpha}\tilde{f}(k). \qquad (3)$$

Riemann-Liouville operator

$$\int\limits_0^{\infty} e^{-\lambda t}\mathrm{D}_{0+}^{\beta}f(t)dt = \lambda^{\beta}\tilde{f}(\lambda). \qquad (4)$$

Solution for point instantaneous source

$$S(\vec{r}, t, E) = S_0 E^{-p} \delta(\vec{r}) \delta(t)$$

$$N(\vec{r}, t, E) = S_0 E^{-p} (D(E, \alpha, \beta) t^{\beta})^{-3/\alpha} \times \\ \times \Psi_3^{(\alpha,\beta)}(|\vec{r}|(D(E, \alpha, \beta) t^{\beta})^{-1/\alpha}). \quad (5)$$

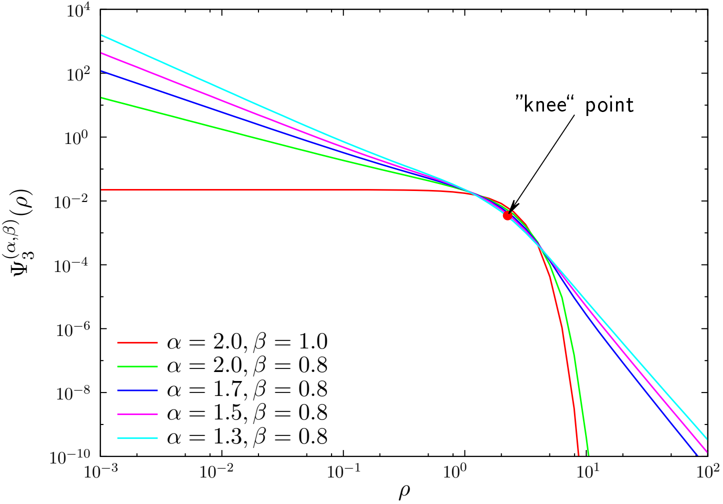
The density of fractional stable distribution $\Psi_3^{(\alpha,\beta)}(\rho)$[1]

$$\Psi_3^{(\alpha,\beta)}(\rho) = \int_0^{\infty} g_3^{(\alpha)} \left(\rho \tau^{\beta/\alpha}\right) g_1^{(\beta,1)}(\tau) \tau^{3\beta/\alpha} d\tau$$

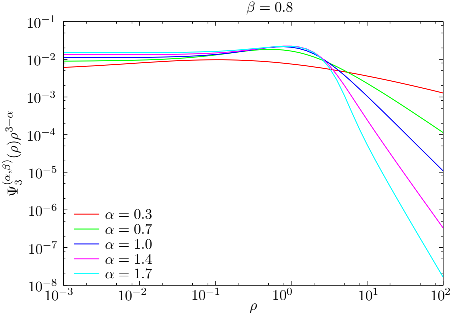
is determined by three-dimensional spherically-symmetrical stable distribution $g_3^{(\alpha)}(\rho)$ $(\alpha \leq 2)$ and one-sided stable distribution $g_1^{(\beta,1)}(t)$ with characteristic exponent $\beta \leq 1$.

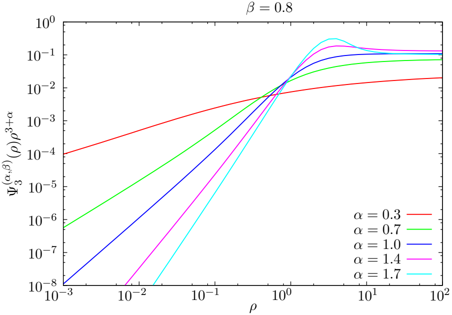
[1] Uchaikin V.V., Zolotarev V.M.: 1999, Chance and stability, VSP. Netherlands, Utrecht.

Three-dimensional density of fractional stable distribution $\Psi_3^{(\alpha,\beta)}(\rho)$ for different values of (α, β)



Asymptotics of $\Psi_3^{(\alpha,\beta)}(\rho)$





$\rho \to 0$

$$\Psi_3^{(\alpha,\beta)}(\rho) \propto \rho^{-(3-\alpha)}$$

$\rho \to \infty$

$$\Psi_3^{(\alpha,\beta)}(\rho) \propto \rho^{-3-\alpha}$$

Knee in the cosmic rays spectrum

Taking into account that $\rho \equiv |\vec{r}|(D(E,\alpha,\beta)t^{\beta})^{-1/\alpha}$, we find

$$N \sim E^{-\eta} = E^{-p+\delta}, \quad E \ll E_k$$

and

$$N \sim E^{-\eta} = E^{-p-\delta}, \quad E \gg E_k,$$

where E_k is the knee energy.

Since $\Delta\eta = \eta|_{>E_k} - \eta|_{<E_k}$ is known from experimental data to be equal $\Delta\eta \sim 0.6$, last equations permit to retrieve self-consistently both spectral exponents p and δ:

$$\delta = \Delta\eta/2 \sim 0.3,$$

$$p = \eta|_{<E_k} + \delta = \eta|_{>E_k} - \delta \approx 2.8 \div 2.9.$$

At $E = E_k$ spectral index $\eta(E_k)$ is equal to injection exponent p at the Galactic source.

Recent results on SNR (Fermi-Lat, H.E.S.S., VERITAS)

- SNR IC443 $\Rightarrow p \approx 2.87$ for $E > 69$ GeV (A.A. Abdo, M. Ackermann, M. Ajello et al. // ApJ, 2010, **712**, 459),
- SNR W44 $\Rightarrow p \approx 3.3$ (A.A. Abdo, M. Ackermann, M. Ajello et al. // Science, 2010, **327**, 1103).
- SNR W49B $\Rightarrow p \approx 2.84$ (H.E.S.S. Collaboration, H. Abdalla, A. Abramowski, F. Aharonian et al. arXiv:1609.00600).
- RX J1713.7-3946 $\Rightarrow p \approx 3.0$ (T. Tanaka, Y. Uchiyama, F. Aharonian et al. // ApJ, 2008, **685**, 988).
- Tycho's SNR $\Rightarrow p \approx 2.92$ (S. Archambault, A. Archer, W. Benbow et al. // ApJ, 2017, **836**, 23).

Cosmic rays spectrum

$$J(\vec{r},t,E) = J_G(\vec{r},E) + J_L(\vec{r},t,E) + J_{NS}(\vec{r},E).$$

Here

- J_G is the global spectrum component determined by the multiple old ($t \geq 10^6$ yr) distant ($r \geq 1$ kpc) sources.
- J_L is the local component, i.e. the contribution of nearby ($r < 1$ kpc) young ($t < 10^6$ yr) sources. The spatial and temporal coordinates of the local sources can be found in our papers.
- J_{NS} is the flux of non-scattered particles.

Cosmic rays spectrum

$$J(\vec{r},t,E) = \frac{v}{4\pi}\Bigg[S_G E^{-p-\delta/\beta} + \frac{S_0 E^{-p}}{D(E,\alpha,\beta)^{3/\alpha}}\times$$

$$\times \sum_{\substack{r_j<1\ \mathrm{kpc}\\ t_j<10^6\ \mathrm{yr}}} \int\limits_{\max[0,t_j-T]}^{t_j} d\tau\tau^{-3\beta/\alpha}\Psi_3^{(\alpha,\beta)}\left(|\vec{r}_j|(D(E,\alpha,\beta)\tau^\beta)^{-1/\alpha}\right) +$$

$$+ S_{NS}\sum_{r_j<1\ \mathrm{kpc}} E^{-p+\delta_L}|\vec{r}_j|^{-\alpha}\Bigg]\exp\left(-\frac{E}{E_0}\right). \quad (6)$$

$$p = 2.85, \quad \delta = 0.27, \quad \delta_L = \delta/2, \quad E_0 = 4\cdot 10^{18} Z\ \mathrm{eV}, \quad T = 10^4\ \mathrm{y}$$

$$\alpha = 1.7$$

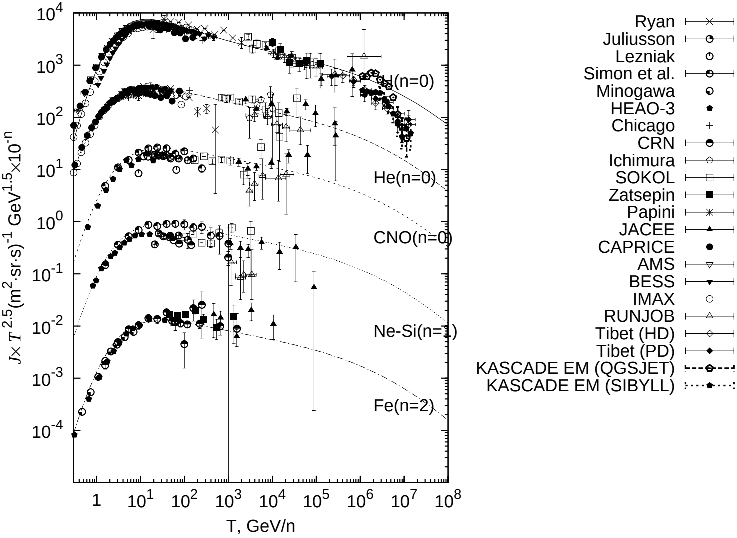
$$\Rightarrow \quad D_0 \approx 1.5\cdot 10^{-3}\mathrm{pc}^{1.7}/\mathrm{yr}^{0.8}$$

$$\beta = 0.8$$

A.A. Lagutin, N.V. Volkov, A.G. Tyumentsev, R.I. Raikin, arXiv:1703.02795

Cosmic rays spectrum in the AD model

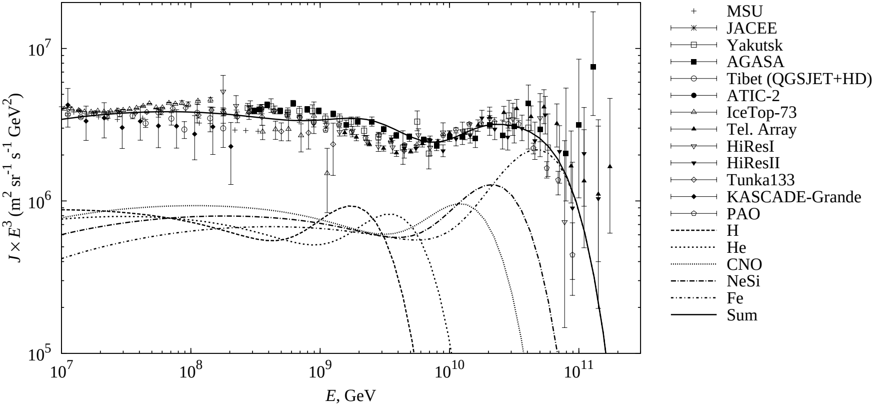
$$S(E) = S_0 E^{-2.85}$$



The knee energies for different elemental groups of nuclei

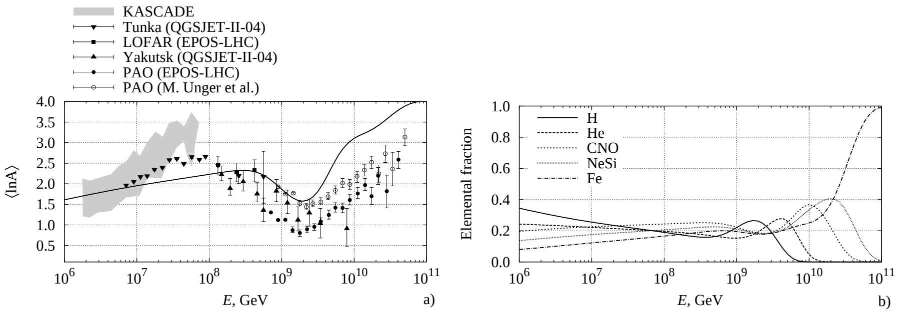
Nuclei	Knee energy, eV
H	$(6.1 \div 6.5) \cdot 10^{14}$
He	$(1.8 \div 2.2) \cdot 10^{15}$
CNO	$(5.8 \div 6.3) \cdot 10^{15}$
NeMgSi	$(0.8 \div 1.2) \cdot 10^{16}$
Fe	$(2.6 \div 3.0) \cdot 10^{16}$

UHECR spectrum in the AD model



Our model contains the multiple ankle-like features in spectra of elemental groups of nuclei at $E > 3 \cdot 10^{17} Z$ eV.

A.A. Lagutin, N.V. Volkov, A.G. Tyumentsev, R.I. Raikin, arXiv:1703.02795



Mean logarithmic mass (a) and elemental fractions (b) of CRs predicted by the anomalous diffusion model.

A.A. Lagutin, N.V. Volkov, A.G. Tyumentsev, R.I. Raikin, arXiv:1703.02795

Mass composition: Basic model predictions

- The composition becomes heavier with energy from the knee to $\sim 10^{17.5}$ eV and reaches a maximum of mean logarithmic mass $\langle \ln A \rangle \sim 2.4$.
- In the energy region $(4 \cdot 10^{17} \div 2 \cdot 10^{18}$ eV) the mean logarithmic mass decreases reaching the minimum value of $\langle \ln A \rangle \sim 1.6$.
- The rapid weighting of the mass composition is observed at $E > 2.5 \cdot 10^{19}$ eV up to the pure iron composition at the cut-off.

Part 2
Normal diffusion of cosmic rays in non-homogeneous ISM

- The medium properties of inhomogeneous ISM vary in space and time.
- The particles emitted by Galactic sources en route to the Solar system pass through the regions of the Galaxy, that have the different diffusion coefficients.
- The particles with probability density $\varphi(D_0)$ propagate in the ISM with a diffusion coefficient D_0,

$$\int\limits_0^\infty \varphi(D_0)dD_0 = 1.$$

Observed density $N(\vec{r}, t, E)$

Homogeneous ISM: Solution of the Ginzburg-Syrovatskii normal diffusion equation

$$N(\vec{r}, t, E) = \frac{S_0 E^{-p}}{(4\pi D_0 E^{\delta} t)^{3/2}} \exp\left(-\frac{r^2}{4 D_0 E^{\delta} t}\right), \tag{7}$$

$$\Downarrow$$

$$dN_D(\vec{r}, t, E) = \frac{S_0 E^{-p} \varphi(D_0) dD_0}{(4\pi D_0 E^{\delta} t)^{3/2}} \exp\left(-\frac{r^2}{4 D_0 E^{\delta} t}\right), \tag{8}$$

$$\Downarrow$$

$$N(\vec{r}, t, E) = \int dN_D(\vec{r}, t, E) = \frac{S_0 E^{-p-3\delta/2}}{(4\pi t)^{3/2}} \times \\ \times \int\limits_0^{\infty} \frac{1}{D_0^{3/2}} \exp\left(-\frac{r^2}{4 D_0 E^{\delta} t}\right) \varphi(D_0) dD_0. \tag{9}$$

Possibles types of diffusivity distribution $\varphi(D_0)$

We follow the approach presented in (S.V. Petrovskii, A.Y. Morozov // Am. Nat., 2009, **173**)
If r is random jump length of particle and τ is the time required to make this jump, the diffusion coefficient is given by equations

$$D_0 = \frac{r^2}{2\tau}, \qquad D_0 = \frac{v^2\tau}{2}$$

same value of r $\swarrow$ $\qquad\qquad$ $\searrow$ same speed v

$$\varphi(D_0) = \psi(\tau)\left|\frac{d\tau(D_0)}{dD_0}\right| = \frac{r^2}{2{D_0}^2}\psi(\tau), \quad \varphi(D_0) = \psi(\tau)\left|\frac{d\tau(D_0)}{dD_0}\right| = \frac{2}{v^2}\psi(\tau).$$

We adopt the following functional form

$$\varphi(D_0) = AD_0^{-\gamma} \exp\left(-\frac{\mathfrak{D}}{D_0}\right), \qquad (10)$$

where $A = \dfrac{\mathfrak{D}^{\gamma-1}}{\Gamma(\gamma-1)}$ is the scaling factor, $\mathfrak{D}$ is the characteristic diffusivity for the Galactic medium, $\gamma > 1$ is an auxiliary parameter, $D_0 > 0$. We additionally define $\varphi(0) = 0$.

$$N(\vec{r}, t, E) = \frac{S_0 A E^{-p-3\delta/2}}{(4\pi t)^{3/2}} \times \times \int\limits_0^{\infty} \frac{dD_0}{D_0^{\gamma+3/2}} \exp\left[-\left(\mathfrak{D} + \frac{r^2}{4E^\delta t}\right)\frac{1}{D_0}\right]. \qquad (11)$$

$$N(\vec{r},t,E) = \frac{S_0 A E^{-p-3\delta/2}}{(4\pi t)^{3/2}} \left(\mathfrak{D} + \frac{r^2}{4E^\delta t}\right)^{-\gamma-1/2} \Gamma\left(\gamma + \frac{1}{2}\right). \qquad (12)$$

At any given moment of time t and for sufficiently large r, so that $r^2 \gg 4\mathfrak{D}E^\delta t$, the density of particles $N(\vec{r},t,E)$ has the form

$$N(\vec{r},t,E) = \frac{S_0 \mathfrak{D}^{\gamma-1}}{\pi^{3/2}(4t)^{1-\gamma}} \frac{\Gamma\left(\gamma+\frac{1}{2}\right)}{\Gamma\left(\gamma-1\right)} \frac{E^{-p-\delta+\delta\gamma}}{r^{2\gamma+1}}$$

or

$$N(\vec{r},t,E) \sim \frac{E^{-p-\delta+\delta\gamma}}{r^{2\gamma+1}}. \qquad (13)$$

The injection spectrum exponent at the Galactic source of the CRs

Thus, instead of the Gaussian distribution of particles density, which is predicted by the homogeneous normal diffusion models, the large-distance asymptotical behavior, we have obtained, is described by a power law $\sim r^{2\gamma+1}$.

AD model

$$N(\vec{r}, t, E) \sim \frac{E^{-p+\delta}}{r^{3+\alpha}}$$

$$\eta|_{<E_k} = p - \delta$$

$$p|_{<E_k} = \eta + \delta$$

Normal diffusion model

$$N(E) \propto E^{-\eta} = E^{-p-\delta+\delta\gamma}$$

$$\eta = p + \delta - \delta\gamma$$

$$p = \eta - \delta(1-\gamma)$$

since $\gamma > 1, p > \eta$.

Model with exponential decrease at $D_0 \to \infty$

It should be noted that a similar anomalous spatial distribution of the particles density may be found in the case of $\varphi(D_0) \sim D_0^{\theta} \exp\left(-\frac{D_0}{\mathfrak{D}}\right)$, where $\mathfrak{D}$ is the characteristic diffusivity for the Galactic medium and $0 < \theta < 1$.

For this scenario we have found

$$N(\vec{r}, t, E) \sim r^{\theta-1} \exp\left(-\frac{r}{\sqrt{\mathfrak{D} E^{\delta} t}}\right) E^{-p-\delta(\theta+1)/2}.$$

Using normal diffusion model, we have shown that the variation of the ISM properties leads to an anomalous spatial distribution of the particles density. Its asymptotic behavior for $1.5 < \gamma < 2$ practically coincides with our result obtained in the framework of the anomalous diffusion model for $1 < \alpha < 2$. In other words, the cosmic rays observed at Earth collectively appear to display non-diffusive (superdiffusive) characteristics although individual particle has moved in a diffusive manner.

We have shown that the average value of the injection index p, obtained in the framework of both transport models, equals to $p \sim (2.8 \div 3.0)$.

Thank you for attention!