NONLINEAR ANALYSES FOR THE DYNAMICS OF SHOCK FRONTS AND DETONATIONS IN GASES

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Detonations = combustion supersonic waves

inert shock followed by an exothermal reaction zone

in gas at ordinary conditions: $1800-3400$ m/s, $15-30$ bar, $2500-3700$ K

Old scientific topic

Recent understanding (nonlinear analyses)

Safety: explosions, nuclear power plant,..

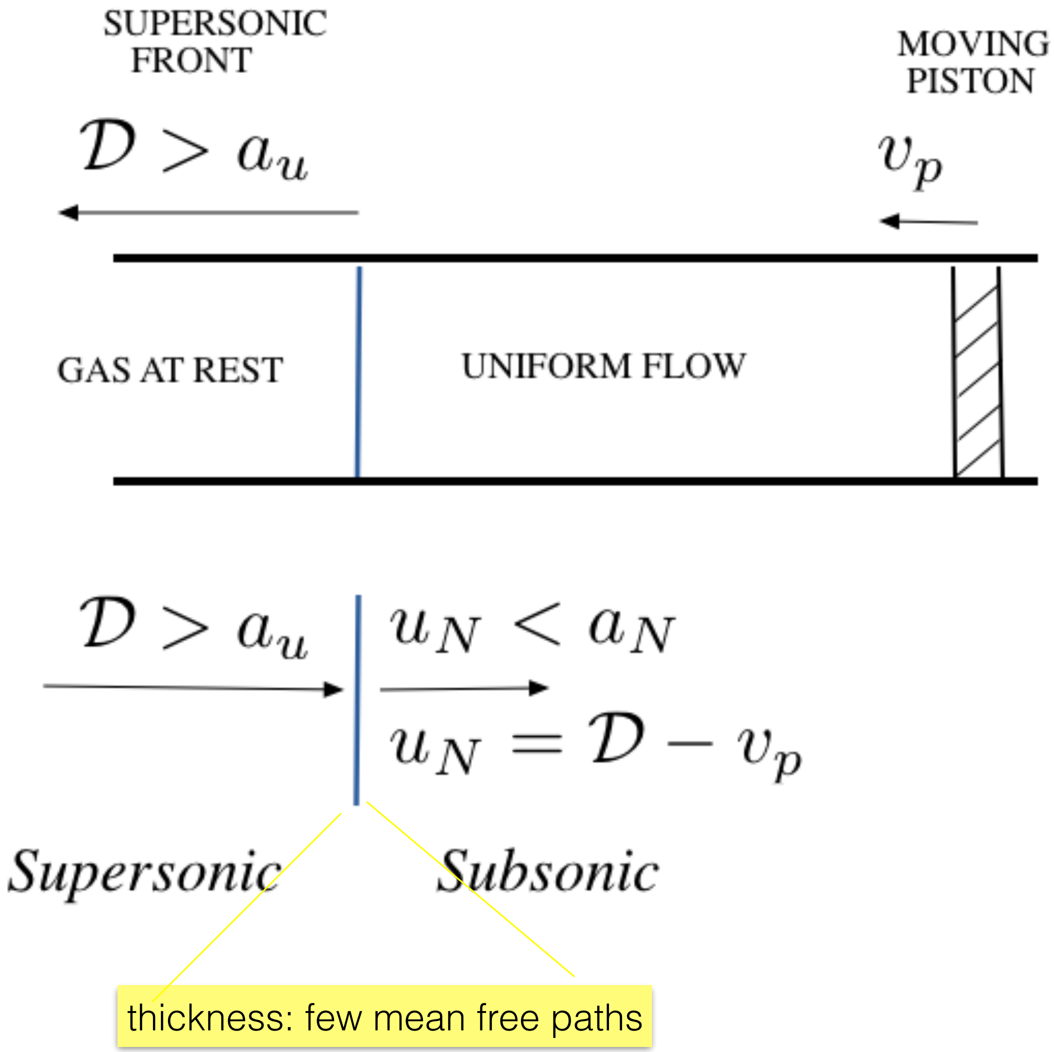
Future propulsion: rotating detonation engines

Astrophysics: explosion of stars, supernovae I

Background in compressible fluids

Planar shock waves and detonations
1860-1940

PLANAR SHOCK WAVE
INERT GAS



Poisson 1808, Stokes 1848, **Riemann** 1860, Rankine 1869, **Hugoniot** 1889, Rayleigh 1910

1-D compressible fluids $u(x,t)$

momentum

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \underbrace{\nu\frac{\partial^2 u}{\partial x^2}}_{\text{viscosity}}$$

mass

$$\frac{1}{\rho}\frac{\partial \rho}{\partial t} + \frac{\partial u}{\partial x} = 0$$

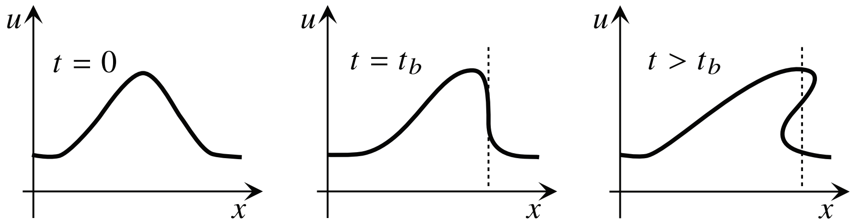
state eq.

$$p(\rho, T)$$

+ energy eq. for T

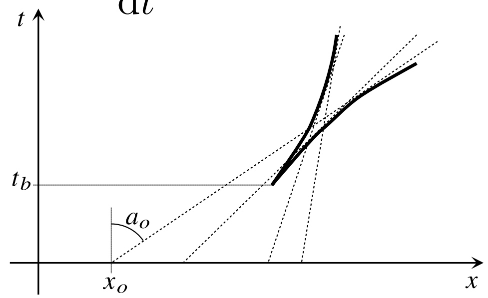
Wave breaking

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$



$t > t_b$: multivalued solution. Wave breaking

$$\frac{\mathrm{d}x}{\mathrm{d}t} = u(x,t)$$



$t > t_b$: characteristics intersect

Burgers equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}$$

Single-valued profile. Stiffening

$u \approx a$ $\nu \approx a\,\lambda$

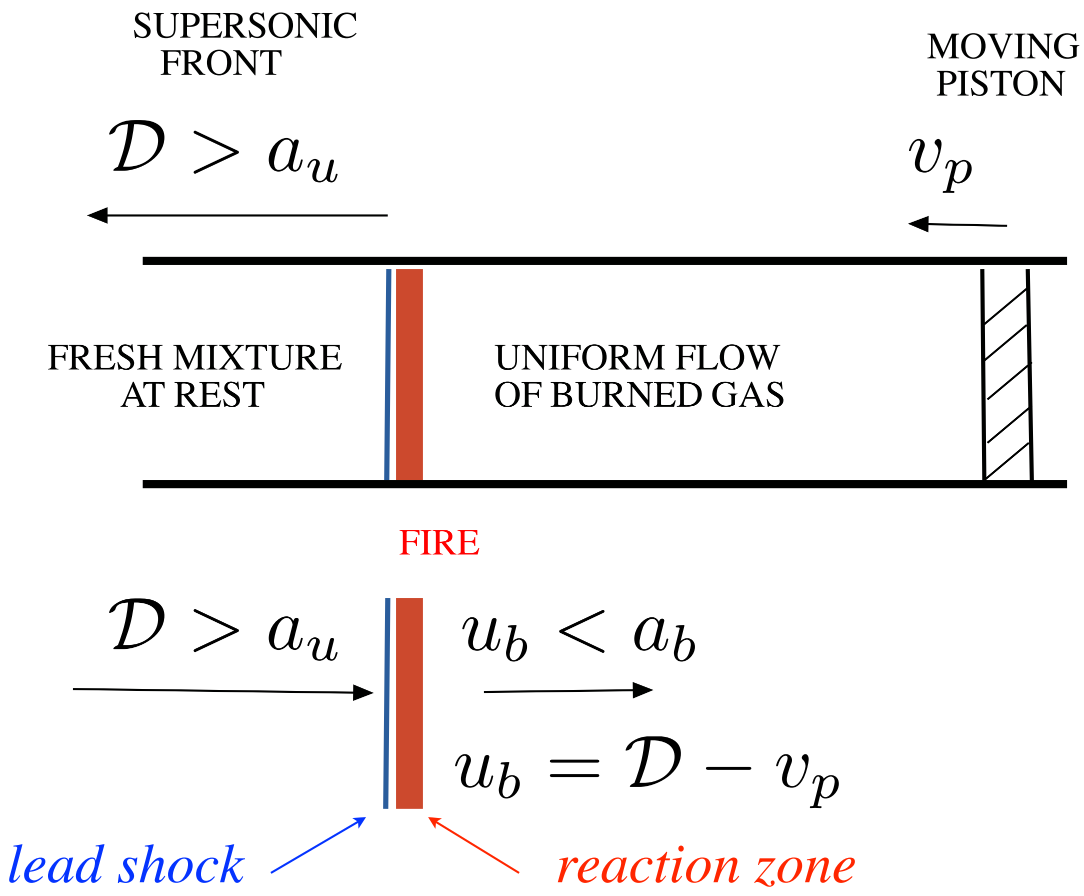
sound speed mean free path

thickness $\approx \lambda$

Boltzmann eq. ?

OVERDRIVEN DETONATION
REACTING GAS

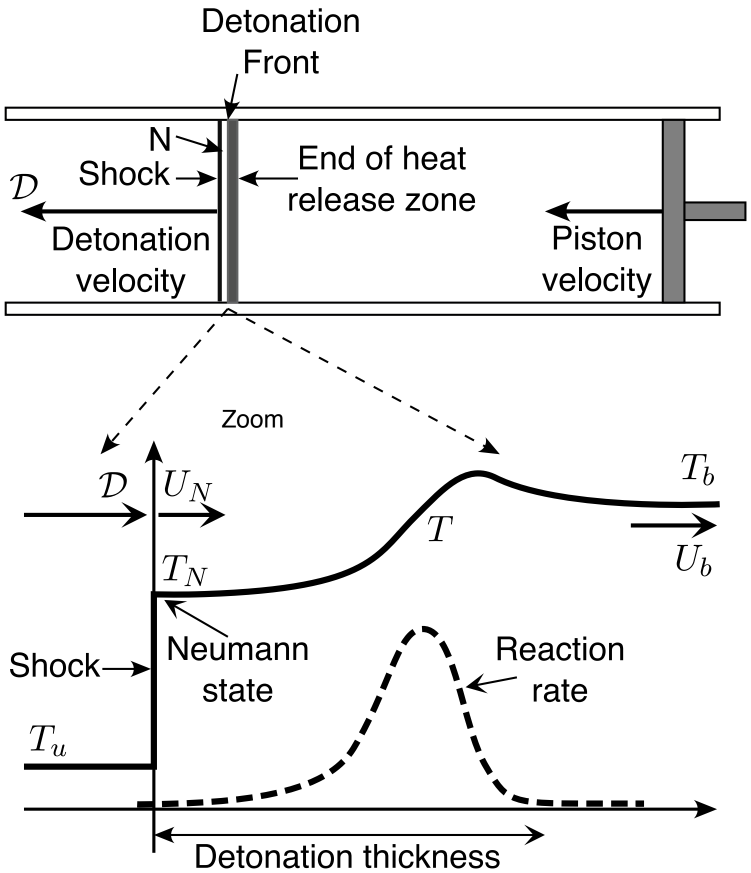
PISTON SUPORTED SUPERSONIC WAVE



Abel 1870, Berthelot et Vielle 1881, Mallard et Le Chatelier 1881, **Mikhel'son** 1893, Chapman 1899, Jouguet 1904,

Vielle 1900, **Zel'dovich** 1940, von Neumann 1942, Döring 1943,

Planar detonation



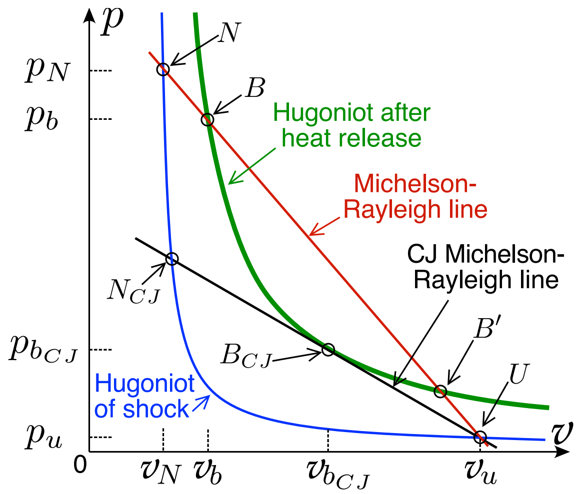
thickness $\approx$ few mms

Self sustained wave

Marginal solution

the so called Chapman-Jouguet wave
(1899) (1905)

Mikhel'son
(1893)



Arrhenius law e^{-E/k_BT}

with a large activation energy

Michelson (1893) Rayleigh (1910)

$$\textit{reaction rate} \approx \frac{e^{-E/k_BT}}{\tau_{coll}}$$

elastic collisions

$$\frac{E}{k_BT} \approx 10$$

$$\mathcal{D}_{CJ} \approx 2000 - 3500 \text{ m/s}$$

Chapman-Jouguet detonation in gases

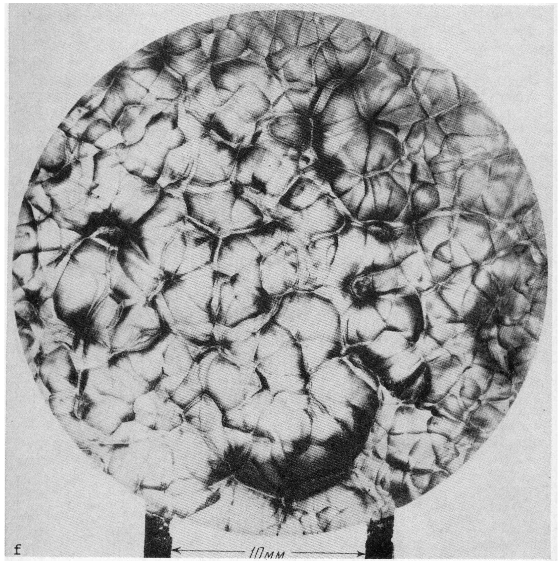
Cellular structure of the front of the detonation waves

experiments & numerics

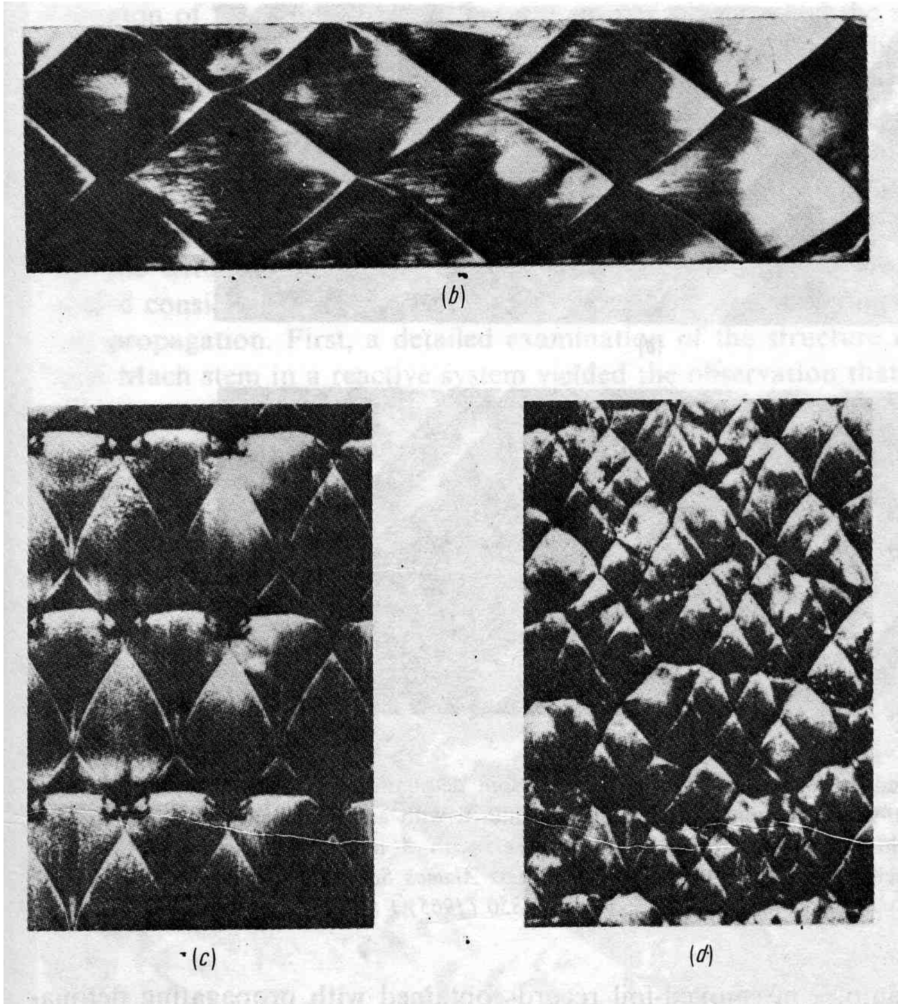
1960-2017

Cellular structure of the detonation wave

Shchelkin and Troshin 1960

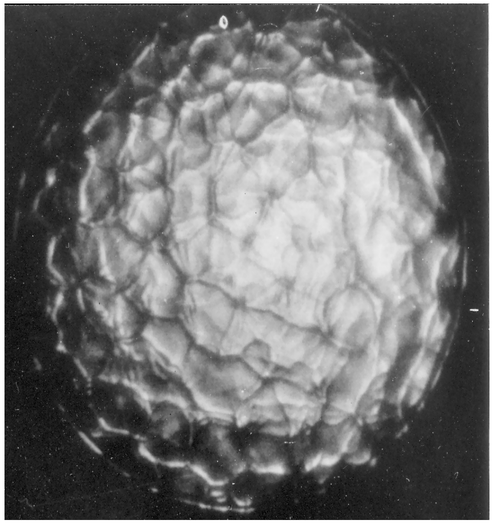


Front view



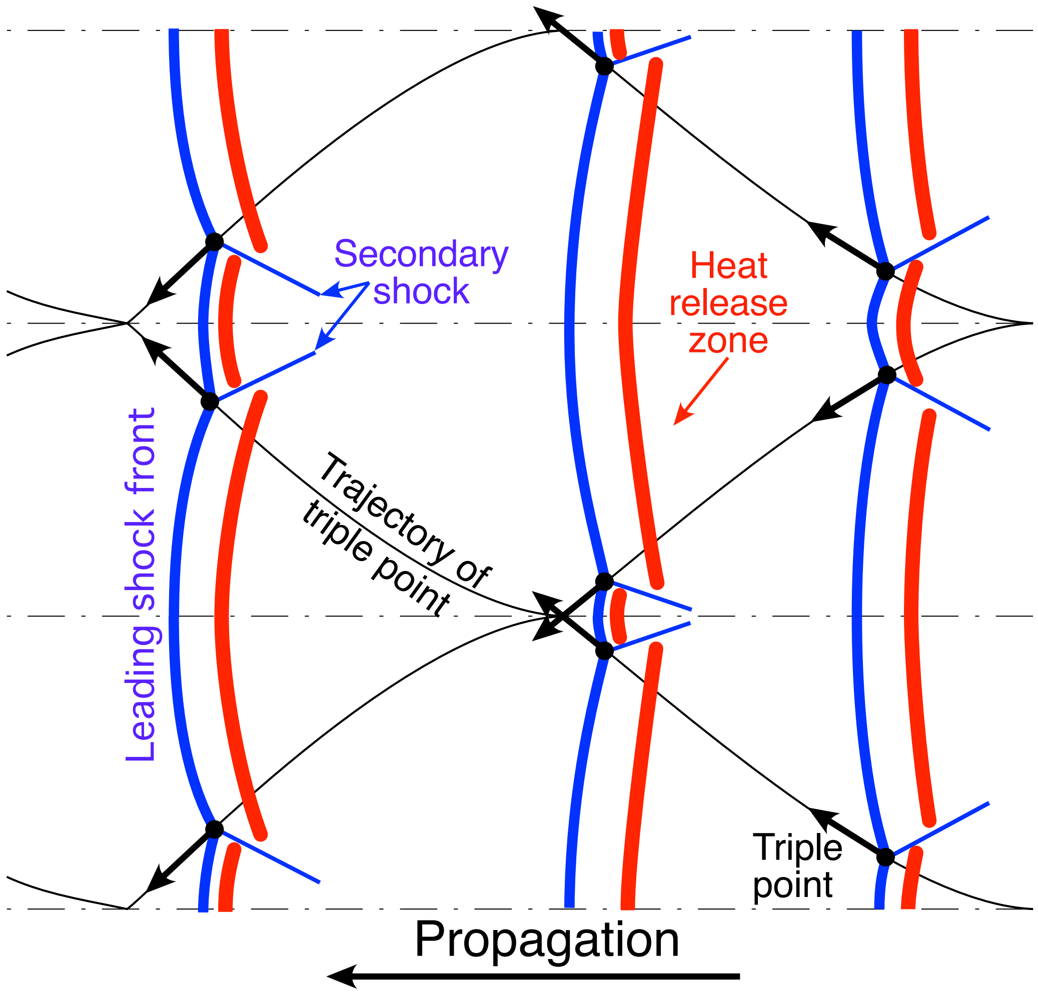
Markings left on soot-coated foils at the wall of the tube

Triple points



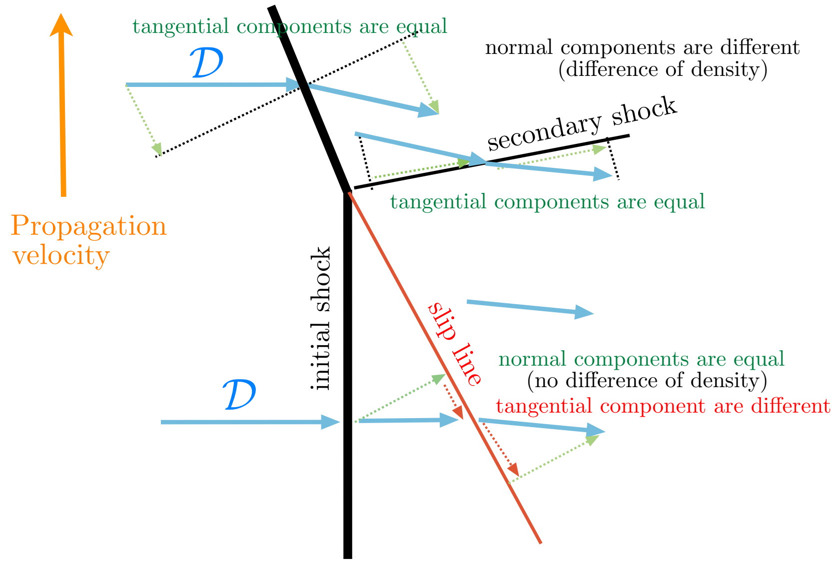
Circular tube 5.2 cm diameter
Presle Desbordes Bauer 1987

$\mathrm{C_2H_2 + 2.5O_2 + 10.5Ar} \qquad p = 0.2\,\mathrm{bar}$

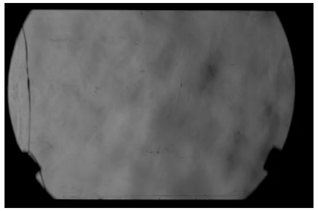


2-D Regular structure

Triple point = 2-D singularity of a shock front



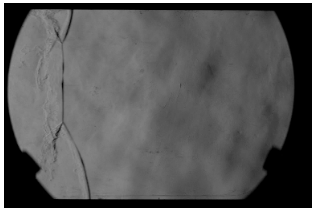
example of a triple point propagating in a uniform flow



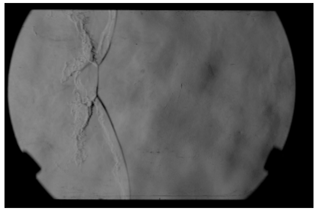
B.Maxwell, R.Bhattacharjee, S. Lau-Chapelaine, S.Falle, G. Sharpe and M. Radulescu

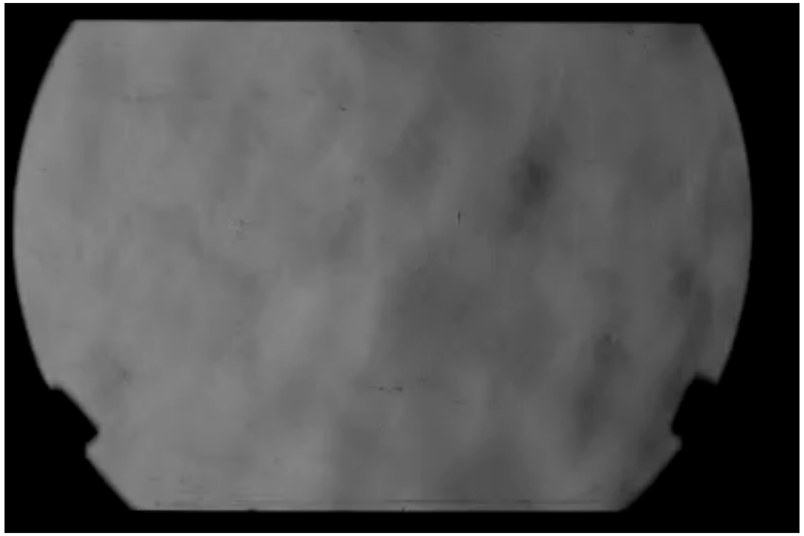
to appear in *Journal of Fluid Mechanics* 2017

$CH_4 + 2O_2 \quad p = 3.5kP_a$ 2D geometry experiment

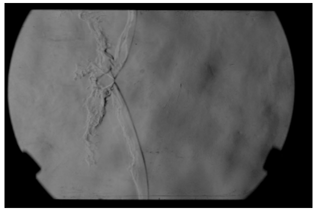


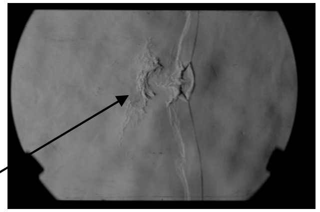
High activation energy: Irregular structure and transient phenomena

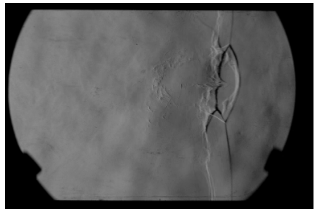


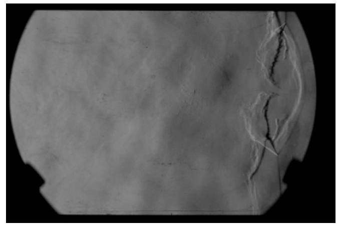


Schlieren images







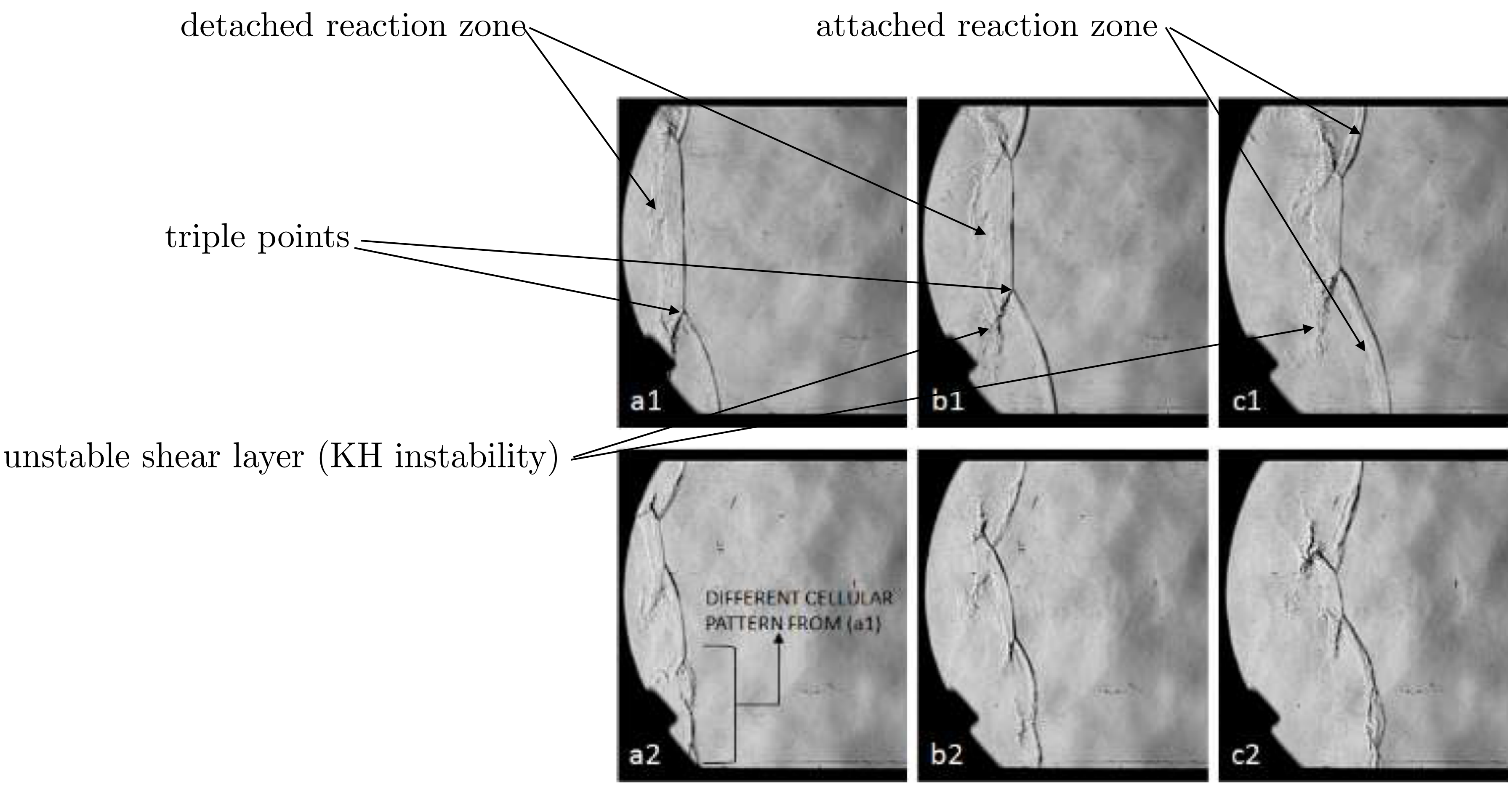


pocket of fresh mixture

Successive frames $11\mu s$ apart

B.Maxwell, R.Bhattacharjee, S. Lau-Chapelaine, S.Falle, G. Sharpe and M. Radulescu
to appear in *Journal of Fluid Mechanics* 2017
$CH_4 + 2O_2 \quad p = 3.5kP_a$

Turbulent mixing of unburnt and burnt pockets ?



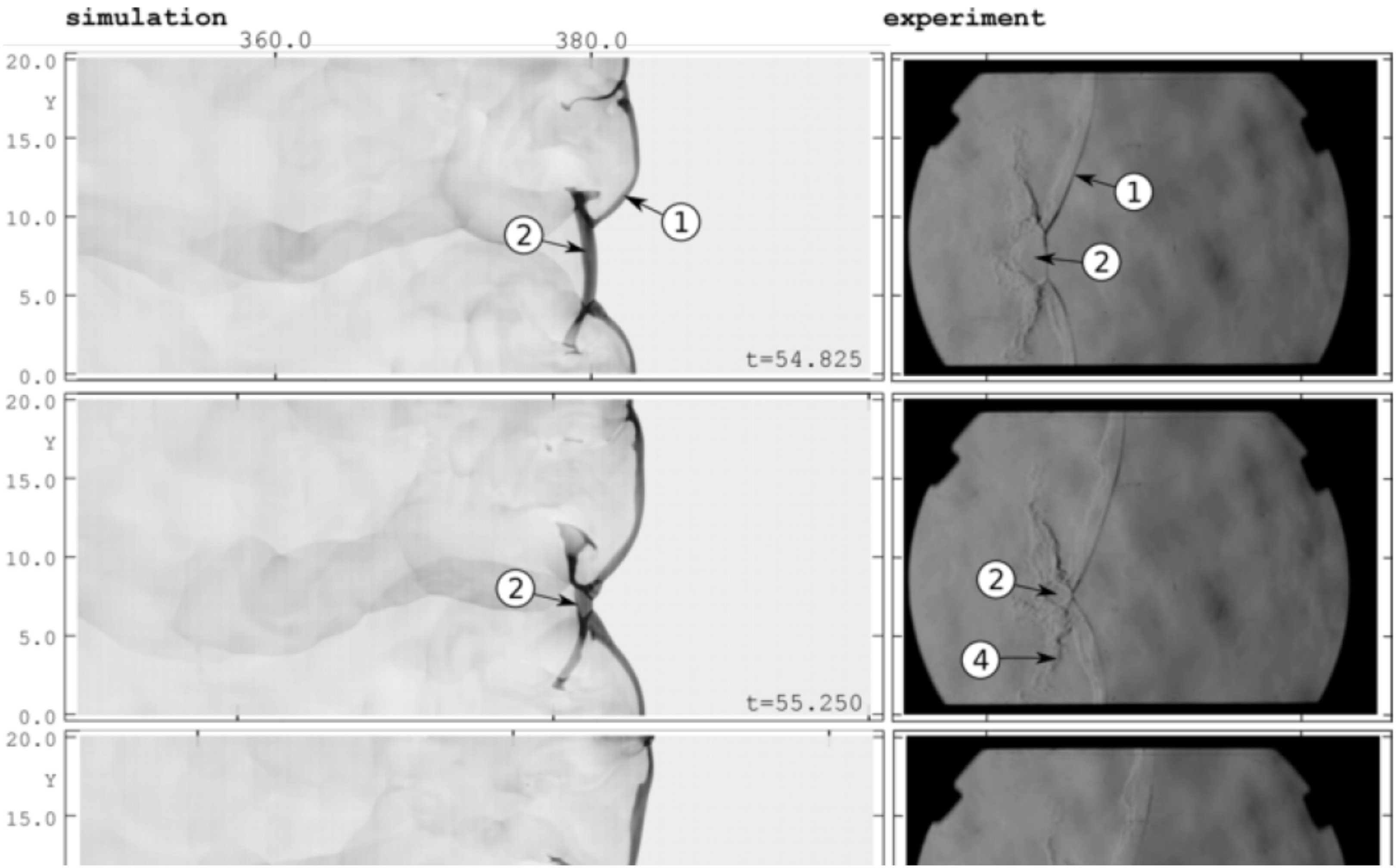
B.Maxwell, R.Bhattacharjee, S. Lau-Chapelaine, S.Falle, G. Sharpe and M. Radulescu

to appear in *Journal of Fluid Mechanics* 2017

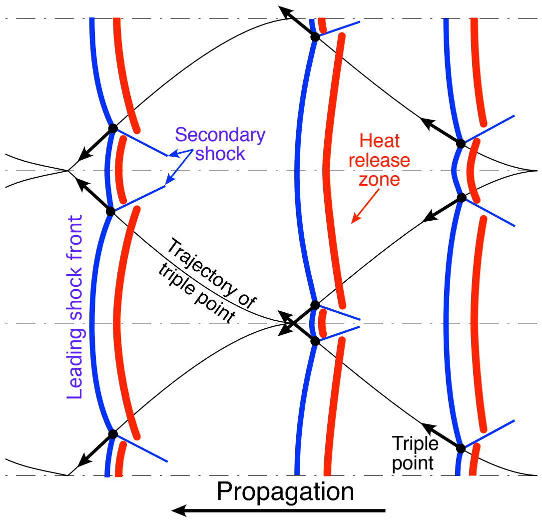
$$CH_4 + 2O_2 \quad p = 3.5kP_a$$

Numerical simulation

Experiment



Theoretical analyses of cellular structures
1996-2017



1-D oscillatory instability + transverse propagation and Mach stem formation

1st step: one-dimensional instability
1996-2002

1-D compressible, inviscid and reactive gas mixture

momentum

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

mass

$$\frac{1}{\rho}\frac{\partial \rho}{\partial t} + \frac{\partial u}{\partial x} = 0$$

state eq.

$$p = (c_p - c_V)\rho T$$

+ energy eq. for T heat release per unit mass progress variable

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right)\left[\ln T - \frac{(\gamma-1)}{\gamma}\ln p\right] = \frac{q_m}{c_p T}\frac{\dot{w}(T,Y)}{t_r}$$

$$\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial r}\right) Y = \frac{\dot{w}(T,Y)}{t_r}$$

entropy wave $\dot{w}$: non-dimensional reaction rate chemical kinetics

hyperbolic form of the equations (characteristics)

acoustic waves heat release rate

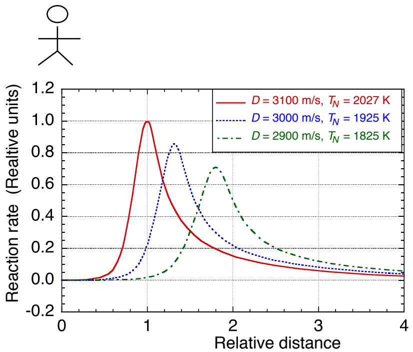
$$\frac{1}{\gamma p}\left[\frac{\partial}{\partial t} + (u \pm a)\frac{\partial}{\partial x}\right] p \pm \frac{1}{a}\left[\frac{\partial}{\partial t} + (u \pm a)\frac{\partial}{\partial x}\right] u = \frac{q_m}{c_p T}\frac{\dot{w}}{t_r}$$

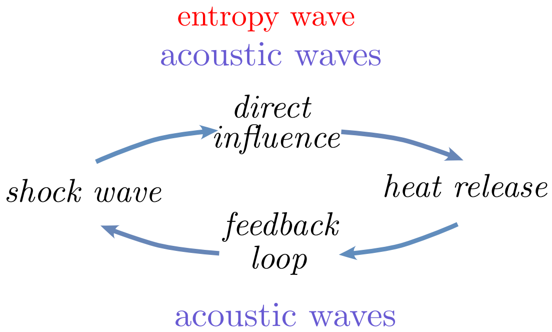
sound speed $a \equiv \sqrt{(\gamma-1)c_p T}$

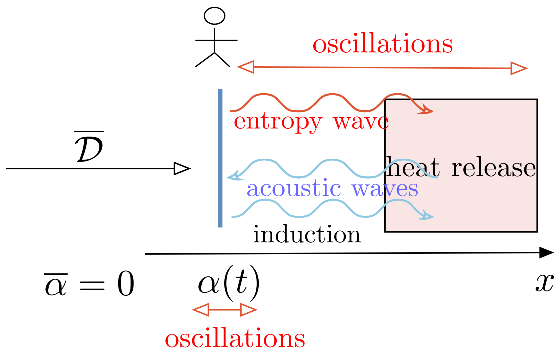
3 nonlinear modes: Entropy wave + 2 acoustic waves

too complicated system for a general analytical solution

1-D longitudinal oscillation of the ZND structure







Analytical solutions have been obtained in $\boxed{\text{two limiting cases}}$ (opposite conditions):

Strongly overdriven detonation in the Newtonian limit (P.C. & L. He 1996)

quasi-isobaric flow

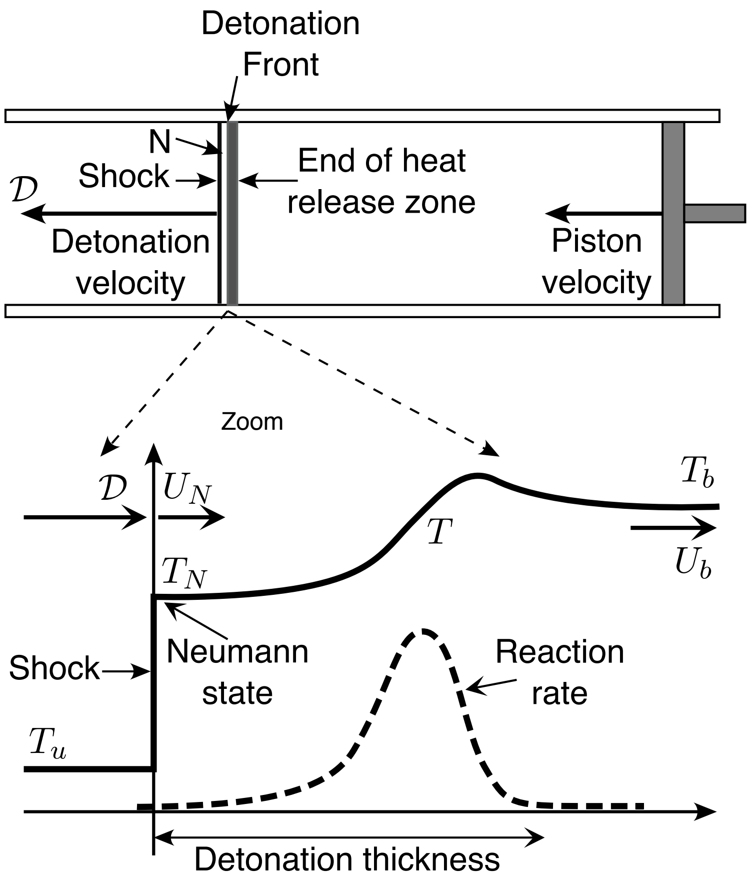
dominant mechanism: entropy wave $\longrightarrow$

CJ (or near CJ) conditions close to the instability threshold (P.C. & F.A. Williams 2002)

transonic flow

dominant mechanism: acoustic wave $\longleftarrow$

Overdriven detonation



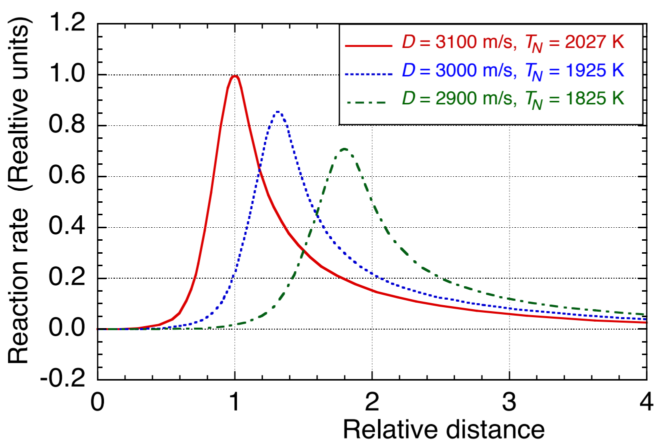
Rankine-Hugoniot relations

detonation velocity shocked gas temperature

$$\mathcal{D} \Leftrightarrow T_N$$

large activation energy

$$\beta \equiv E/k_b T_N \gg 1$$



large sensitivity of the distribution of reaction rate to temperature

steady state: $\Omega(\Theta_N, \mathrm{x})$

$$\Theta_N \equiv \beta \frac{(T_N - \overline{T}_N)}{\overline{T}_N} = O(1)$$

$M_u \equiv \mathcal{D}/a_u$ $\quad M_N \equiv \boxed{u_N/a_N}$

initial fluid | shocked fluid, Neumann state

$\mathcal{D} > a_u$ $\quad u_N < a_N$

$M_u > 1$ $\quad M_N < 1$

(p_u, ρ_u) $\quad (p_N, \rho_N)$

Rankine-Hugoniot

$$M_N^2 = \frac{(\gamma-1)M_u^2 + 2}{2\gamma M_u^2 - (\gamma-1)}$$

$$\gamma \equiv c_p/c_V$$

strongly overdriven detonations in the Newtonian limit

(P.C. & L. He 1996)

$$\boxed{\epsilon^2 \equiv \frac{1}{M_u^2} \ll 1 \qquad (\gamma-1) = O(\epsilon^2) \qquad M_N = O(\epsilon)}$$

shocked gas: quasi-isobaric approximation

low Mach number: $M_N \ll 1$

Rankine-Hugoniot

$$\frac{|\delta p_N|}{\rho_N a_N |\dot{\alpha}_t|} = O(M_N), \quad \delta u_N \approx \dot{\alpha}_t \Rightarrow \boxed{\frac{|\delta p_N|}{\rho_N a_N |\delta u_N|} = O(M_N)}$$

negligible pressure effects; acoustic waves quasi-instantaneous

strongly overdriven detonations in the Newtonian limit

quasi-isobaric approximation in the shocked gases

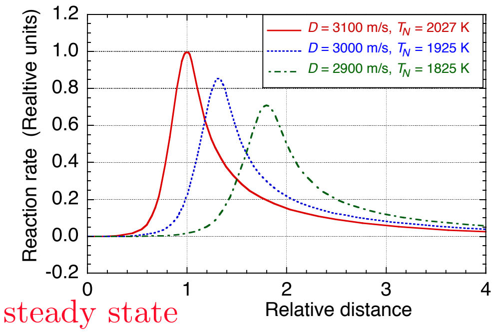
+ energy eq. for T heat release per unit mass progress variable

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}\right)\left[\ln T-\cancel{\frac{(\gamma-1)}{\gamma}}\ln p\right]=\frac{q_m}{c_pT}\frac{\dot{w}(T,Y)}{t_r} \qquad \left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}\right)Y=\frac{\dot{w}(T,Y)}{t_r}$$

$\dot{w}$: non-dimensional reaction rate chemical kinetics

mass

$$\boxed{\frac{1}{\rho}\frac{\partial \rho}{\partial t}+\frac{\partial u}{\partial x}=0} \qquad \boxed{\rho T\approx \text{cst.}}$$



steady state

unsteady distribution of the rate of heat release

$$\boxed{\Omega(\Theta_N(\mathrm{t}-\mathrm{x}),\mathrm{x})}$$

$$\Theta_N(\mathrm{t})=\frac{\beta(T_N(\mathrm{t})-\overline{T}_N)}{\overline{T}_N}$$

$$\mathrm{x}\equiv\frac{1}{t_r\rho_u\overline{\mathcal{D}}}\int_{\alpha(t)}^{x}\rho\,\mathrm{d}x \qquad \mathrm{t}\equiv\frac{t}{t_r}$$

$$\Omega(\Theta_N,\mathrm{x})\approx \mathrm{e}^{\Theta_N}\overline{\Omega}(\mathrm{e}^{\Theta_N}\mathrm{x}), \qquad l/\overline{l}=\mathrm{e}^{-\Theta_N} \qquad \int_0^\infty \Omega(\Theta_N,\mathrm{x})\mathrm{dx}=1$$

Arrhenius

+

conservation of mass and boundary conditions

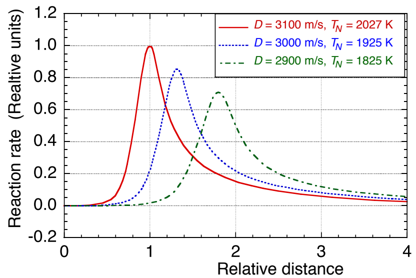
Rankine-Hugoniot at $\mathrm{x}=0$: $\dfrac{\mathcal{D}(\mathrm{t})-\overline{\mathcal{D}}}{\overline{\mathcal{D}}}\Leftrightarrow\Theta_N(\mathrm{t})$ $\qquad$ $\mathrm{x}\longrightarrow\infty$: boundedness

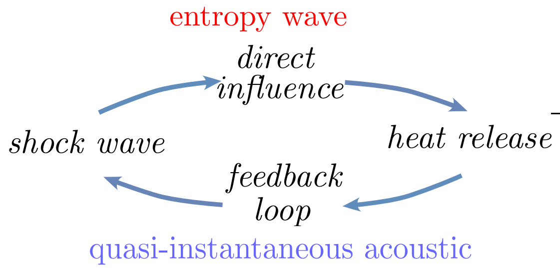
$\Downarrow$

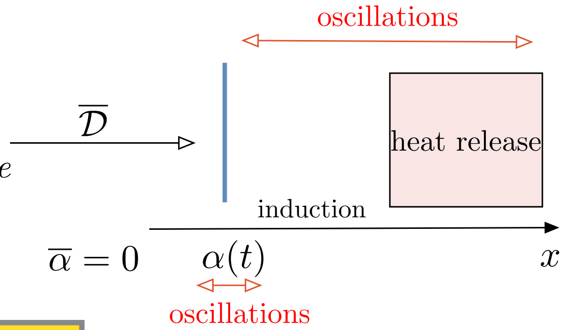
integral equation for $\Theta_N(t)$

$$\boxed{1+b\Theta_N(t)=\int_0^\infty \Omega(\Theta_N(\mathrm{t}-\mathrm{x}),\mathrm{x})\mathrm{dx},} \qquad b^{-1}\equiv\beta_N(\gamma-1)\frac{q_m}{c_p\overline{T}_N}$$

strongly overdriven detonations in the Newtonian limit
quasi-isobaric approximation in the shocked gas



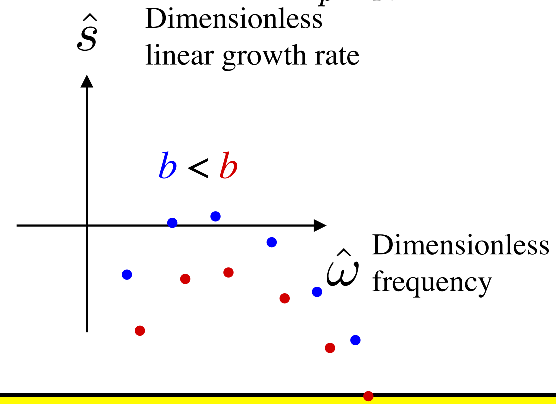




$$\Omega(\Theta_N, \mathrm{x}) \approx \mathrm{e}^{\Theta_N}\overline{\Omega}(\mathrm{e}^{\Theta_N}\mathrm{x})$$

$$\boxed{1 + b\Theta_N(t) = \int_0^\infty \Omega(\Theta_N(\mathrm{t}-\mathrm{x}), \mathrm{x})\mathrm{dx},}$$

$$b^{-1} \equiv \beta_N(\gamma - 1)\frac{q_m}{c_p\overline{T}_N}$$



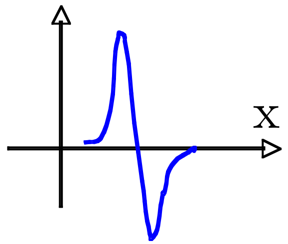
Linearization. Normal mode analysis

$\Theta_N(t) \propto \mathrm{e}^{\sigma t}$

$\sigma = \hat{s} + \mathrm{i}\hat{\omega}$

$\Omega'_N(\mathrm{x}) \equiv \mathrm{d}[\mathrm{x}\overline{\Omega}(\mathrm{x})]/\mathrm{dx}$

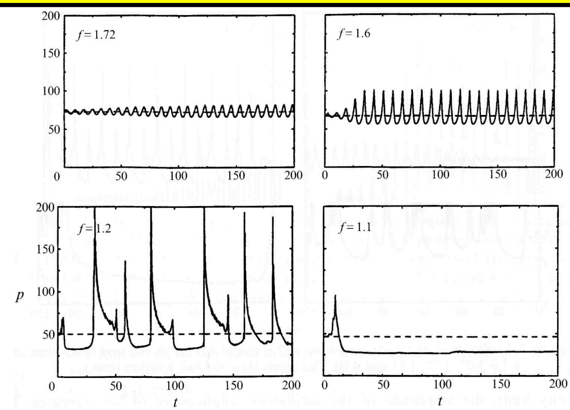
$$\boxed{b = \int_0^\infty \Omega'_N(\mathrm{x})\mathrm{e}^{-\sigma \mathrm{x}}\mathrm{dx}}$$



$\hat{\omega} = O(1) \Leftrightarrow \omega = O(\bar{t}_N)$

frequence of oscillation of order of the transit time

Poincaré-Andronov (Hopf) bifurcation

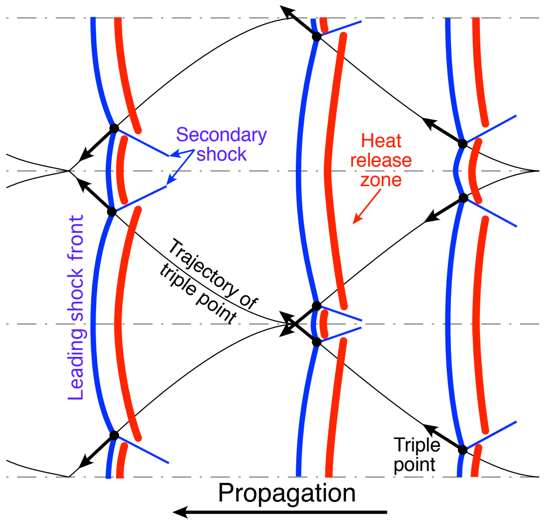


OK with DNS

high thermal sensitivity, β_N
and/or
stiffness of the distribution of heat release $\Omega(\Theta_N, \mathrm{x})$

promote the instability

nonlinear effects: stochasticity + dynamical quenching



formation of singularities propagating in the transverse direction

Nonlinear dynamics of inert shock fronts

P. Clavin, *Journal of Fluid Mechanics* (2013)

B. Denet, K. Biamo, G. Lodato, L. Vervisch and P. Clavin, *Combust. Sci. Technol.* (2014)

G. Lodato, L. Vervisch and P. Clavin, *Journal of Fluid Mechanics* (2016)

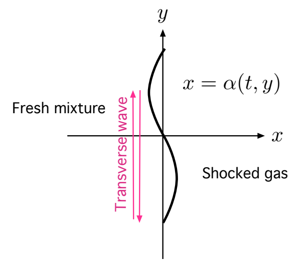
P. Clavin, *Combust. Sci. Technol.* (2017)

$M_u \equiv \mathcal{D}/a_u$ — initial fluid — $\mathcal{D} > a_u$ — $M_u > 1$ — $(p_u,\, \rho_u)$

$M_N \equiv u_N/a_N$ — shocked fluid, Neumann state — $u_N < a_N$ — $M_N < 1$ — $(p_N,\, \rho_N)$

Linear dynamics of shock waves

D'Yakov Kontorovich 1954-57 Clavin Williams 2012 Bates 2012



$\mathrm{D}/\mathrm{D}t = \partial/\partial t + \overline{u}_N \partial/\partial x$

$\partial p/\partial \rho|_{s=\mathrm{cst}} \equiv a$

$$\frac{1}{\overline{\rho}_N}\frac{\mathrm{D}}{\mathrm{D}t}\delta\rho + \frac{\partial}{\partial x}\delta u + \frac{\partial}{\partial y}\delta w = 0,$$

$$\overline{\rho}_N \frac{\mathrm{D}}{\mathrm{D}t}\delta u = -\frac{\partial}{\partial x}\delta p, \qquad \overline{\rho}_N \frac{\mathrm{D}}{\mathrm{D}t}\delta w = -\frac{\partial}{\partial y}\delta p,$$

$$\boxed{\frac{\mathrm{D}}{\mathrm{D}t}\delta s = 0} \Rightarrow \frac{\mathrm{D}}{\mathrm{D}t}\delta p = \overline{a}_N^2 \frac{\mathrm{D}}{\mathrm{D}t}\delta\rho,$$

$$\boxed{\frac{\mathrm{D}}{\mathrm{D}t}(\nabla \times \delta\mathbf{u}) = 0}$$

eliminating δu and δw $\Rightarrow$ $\boxed{\frac{\mathrm{D}^2}{\mathrm{D}t^2}\delta p - \overline{a}_N^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\delta p = 0}$

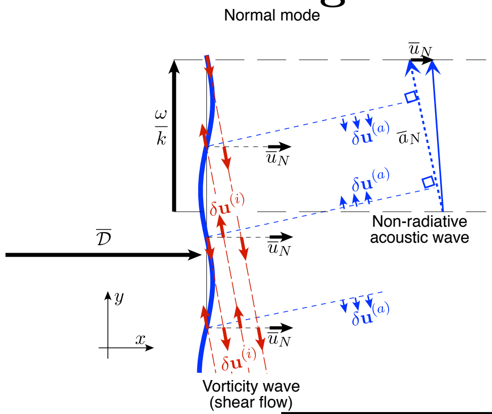
linear modes in the shocked gas: acoustic waves + entropy vorticity wave

+

Rankine-Hugoniot conditions at the shock front $x = \alpha(y, t)$ & bc at $x \to \infty$

Result for a polytropic gas :

Oscillatory mode with **a non-radiating condition** of the acoustic wave



Laplace transform $\Rightarrow$ stability with a power law relaxation $1/t^{2/3}$

Strong shock in the Newtonian limit

$$\boxed{\epsilon^2 \equiv \frac{1}{M_u^2} \ll 1 \qquad (\gamma - 1) = O(\epsilon^2)}$$

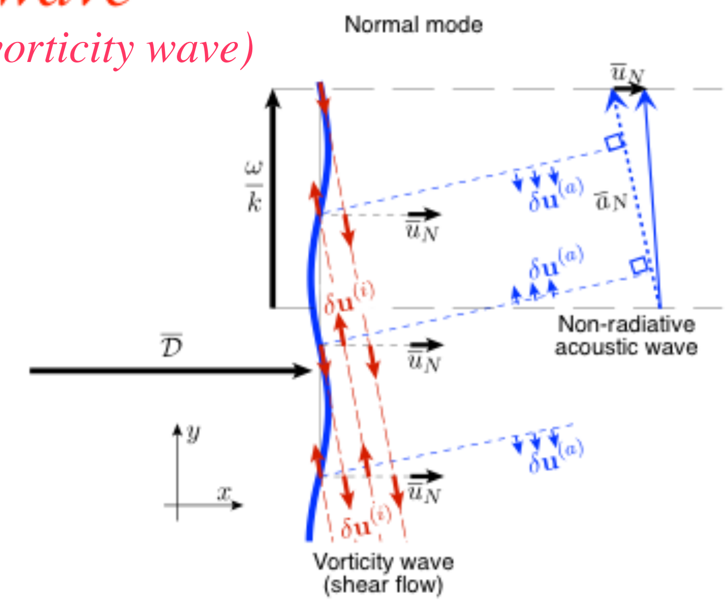
$$M_N^2 = O(\epsilon^2)$$

The flow of compressed gas in the normal modes
is dominated by the shear wave

(entropy-vorticity wave)

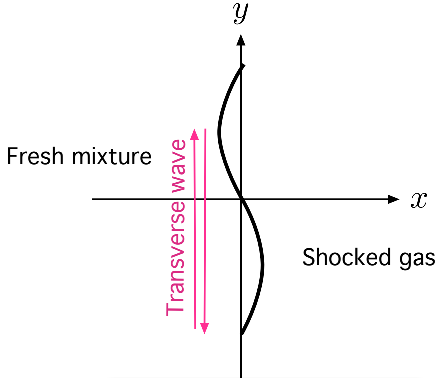
$$\delta u = \delta u^{(i)} + \delta u^{(a)} \qquad \delta w = \delta w^{(i)} + \delta w^{(a)}$$

$$\boxed{|\delta u^{(a)}/\delta u^{(i)}| = O(M_N^2) \qquad |\delta w^{(a)}/\delta w^{(i)}| = O(M_N^2)}$$



$$\epsilon^2 \equiv \frac{1}{M_u^2} \ll 1 \qquad (\gamma - 1) = O(\epsilon^2)$$

quasi-isobaric approximation



$$\boxed{x = \alpha(y,t)}$$

$$\left(\frac{\partial}{\partial t} + u_N \frac{\partial}{\partial x}\right)\delta u^{(i)} = 0 \qquad x = 0: \quad \delta u^{(i)} = \dot{\alpha}_t \qquad \dot{\alpha}_t \equiv \partial\alpha/\partial t$$

$$\left(\frac{\partial}{\partial t} + u_N \frac{\partial}{\partial x}\right)\delta w^{(i)} = 0 \qquad x = 0: \quad \delta w^{(i)} = \mathcal{D}\alpha'_y \qquad \alpha'_y \equiv \partial\alpha/\partial y$$

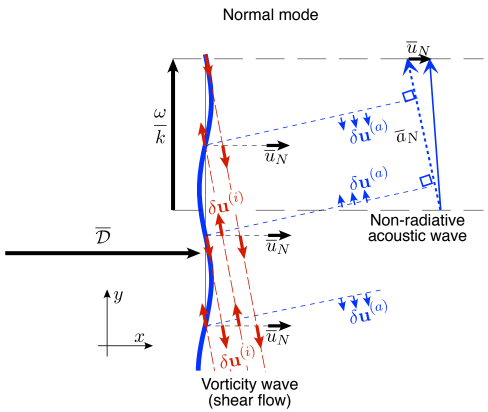
Rankine-Hugoniot conditions

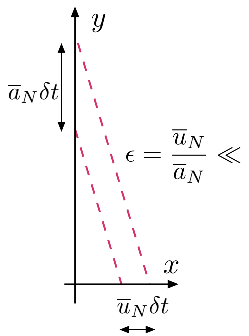
$$\boxed{\begin{aligned}\delta u^{(i)}(x,y,t) &= \dot{\alpha}_t(y, t - x/u_N)\\ \delta w^{(i)}(x,y,t) &= \mathcal{D}\alpha'_y(y, t - x/u_N)\end{aligned}}$$

incompressibility $\quad \dfrac{\partial}{\partial x}\delta u^{(i)} + \dfrac{\partial}{\partial y}\delta w^{(i)} = 0$ *with* $\quad \dfrac{\partial}{\partial x}\delta u^{(i)} = -\dfrac{1}{u_N}\dfrac{\partial}{\partial t}\delta u^{(i)}$

$M_u \equiv \mathcal{D}/a_u$	$M_N \equiv u_N/a_N$
initial fluid	shocked fluid, Neumann state
$\mathcal{D} > a_u$	$u_N < a_N$
$M_u > 1$	$M_N < 1$
$(p_u,\ \rho_u)$	$(p_N,\ \rho_N)$

$$\boxed{u_N\mathcal{D} \approx a_N^2} \quad \Rightarrow \quad \boxed{\frac{\partial^2\alpha}{\partial t^2} - a_N^2\frac{\partial^2\alpha}{\partial y^2} = 0}$$



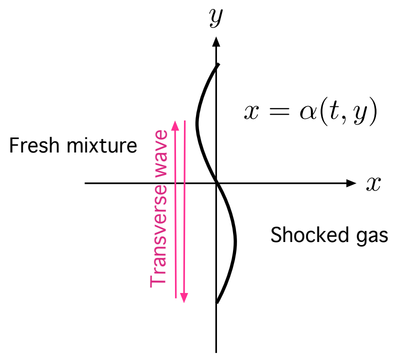


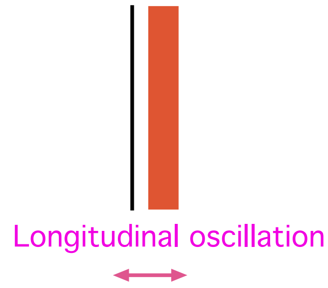
$$\epsilon^2 \equiv \frac{1}{M_u^2} \ll 1 \qquad (\gamma - 1) = O(\epsilon^2) \quad \Longrightarrow \quad M_N = O(\epsilon)$$

Strong shock in the Newtonian limit

initial fluid	shocked fluid, Neumann state
$\mathcal{D} > a_u$	$u_N < a_N$
$M_u > 1$	$M_N < 1$
$(p_u,\, \rho_u)$	$(p_N,\, \rho_N)$

Order of magnitude of the cell size





$$\omega \approx a_N k \qquad\qquad \omega \approx 2\pi/\tau_N, \quad (\tau_N \equiv \tau_{reac}(T_N))$$

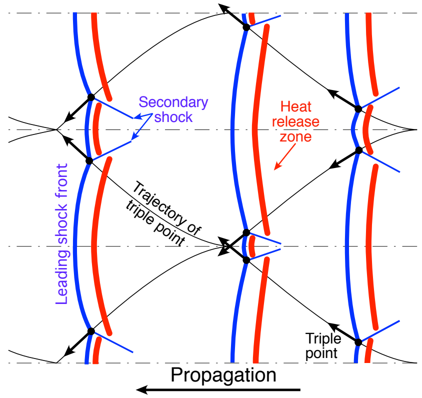
$$k \equiv 2\pi/\Lambda \approx 2\pi/(\tau_N a_N) \qquad\qquad \Lambda \approx a_N \tau_N$$

detonation thickness $\boxed{d_N \approx u_N \tau_N}$ $\quad \boxed{\Lambda \approx d_N / M_N}$

$$\boxed{\Lambda / d_N \approx 1/M_N \gg 1}$$

markings left on sooted coil foils at the wall

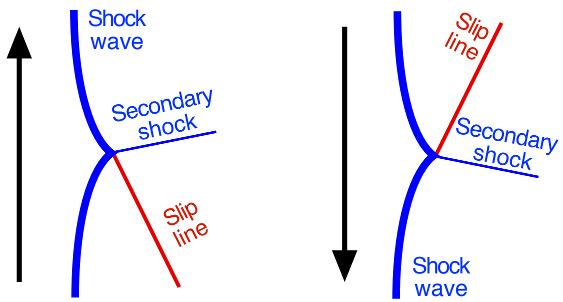
trajectory of triple points



Nonlinear effects: formation of triple points (Mach stems)

pure (inert) shock wave

Mach stems propagating
in the transverse direction

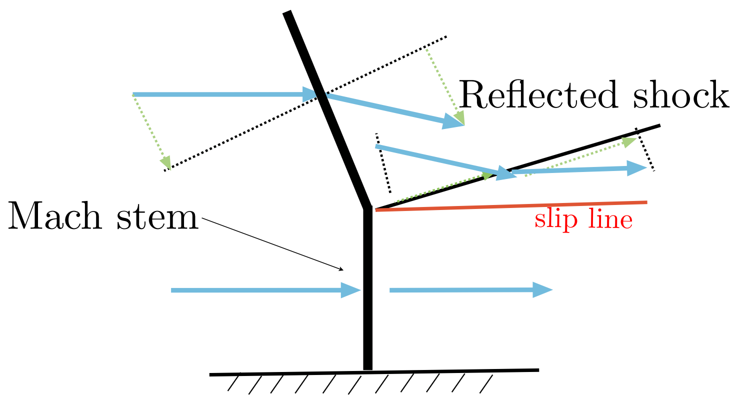


What is a Mach stem ?

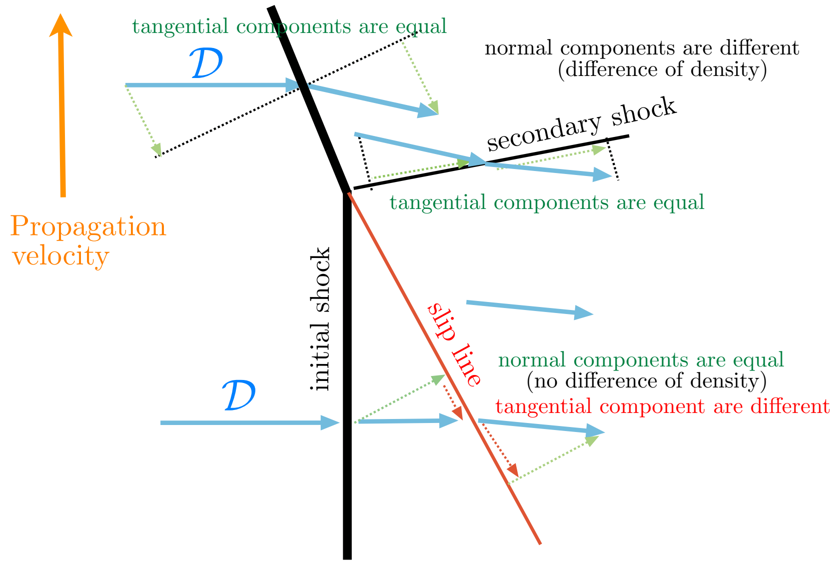
Triple point = 3 shock waves + 1 slip line (degenerescence of shear layer)

called also contact line discontinuity (Courant Friedrichs 1948)

steady triple points were
first observed as a reflexion of an oblique incident shock at a wall



example of stationary triple point



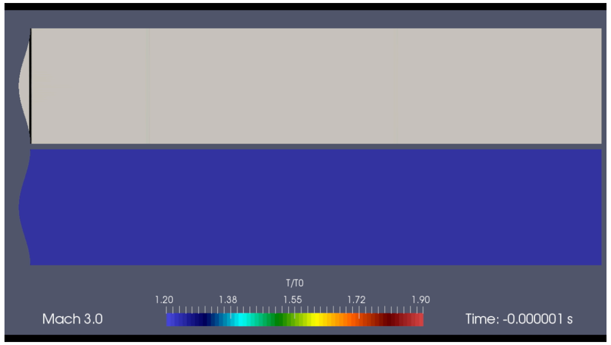
example of a triple point propagating in a uniform flow

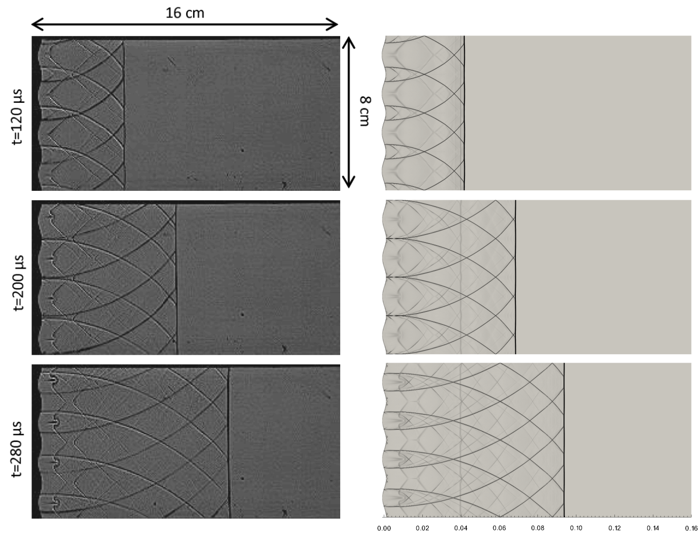
Spontaneous formation of Mach stems on shock fronts

Shock reflection from a wavy wall

2-D Direct Numerical Simulation

Lodato et al. *J.F.M* (2016)





Experiments: Schlieren

Denet et al. *C.S.T* (2015)

D.N.S

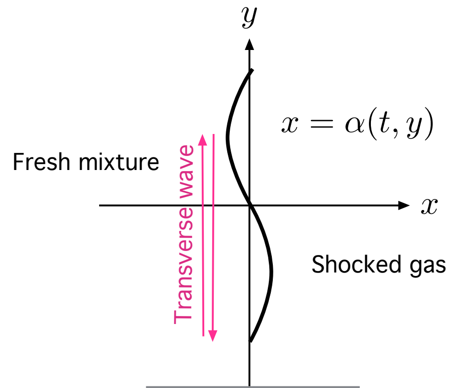
Structure similar to that of cellular detonation

Weakly nonlinear analysis of the shock dynamics (strong shocks in the Newtonian limit)

$$\epsilon^2 \equiv \frac{1}{M_u^2} \ll 1 \qquad (\gamma - 1) = O(\epsilon^2) \quad \Rightarrow \quad M_N = O(\epsilon)$$

$$x = \alpha(y, t)$$

notation : $\alpha'_y(y, t) \equiv \partial\alpha/\partial y$



Attention is limited to small nonlinear corrections: $\boxed{|\alpha'_y|/\epsilon \ll 1}$

Rankine-Hugoniot condition

boundary conditions: $\quad x = 0 : \quad u = \overline{u}_N + \dot{\alpha}_t + \boxed{\overline{\mathcal{D}}\alpha'^2_y}$

nonlinearity

$$\boxed{\frac{\partial^2\alpha}{\partial t^2} - \overline{a}_N^2 \frac{\partial^2\alpha}{\partial y^2} + \overline{\mathcal{D}} \frac{\partial}{\partial t}\left(\frac{\partial\alpha}{\partial y}\right)^2 = 0}$$

Formation of singularities. Simple transverse wave: Burgers equation for $\alpha'_y(y, t)$

Formation of a singularity in a finite time

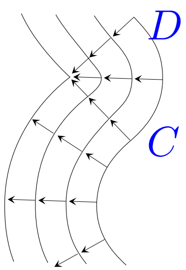
$$\frac{\partial^2 \alpha}{\partial t^2} - \overline{a}_N^2 \frac{\partial^2 \alpha}{\partial y^2} + \mathcal{D} \frac{\partial}{\partial t} \left(\frac{\partial \alpha}{\partial y} \right)^2 = 0$$

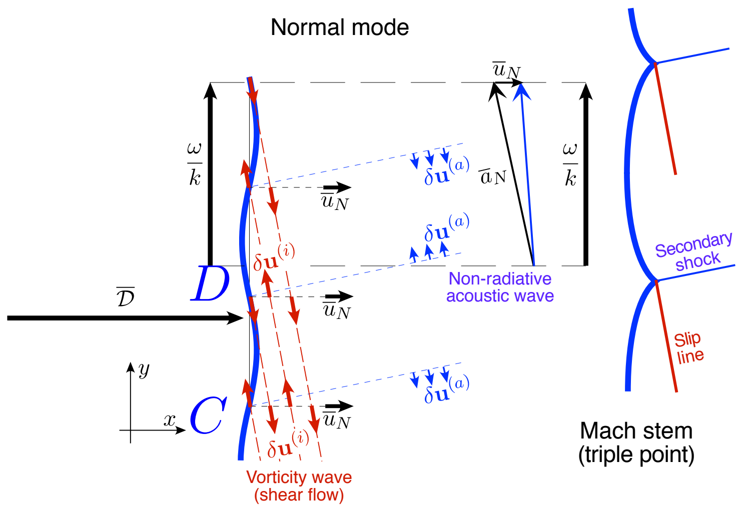
Burgers equation for $\alpha'_y(y,t) \equiv \partial\alpha/\partial y$ along a simple transverse wave

$$\left[\frac{\partial}{\partial t} \pm \overline{a}_N \frac{\partial}{\partial y} \right] \alpha'_y + 2\mathcal{D} \left(\alpha'_y \frac{\partial}{\partial y} \alpha'_y \right) = 0$$

Geometrical construction

$x = 0: \quad u = \overline{u}_N + \dot{\alpha}_t + \boxed{\overline{\mathcal{D}} \alpha'^2_y}$ nonlinearity





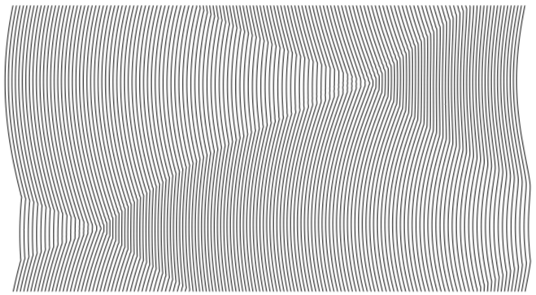
nonlinear term $\Rightarrow$ Collapse of points C and D by a Huygens construction

shear wave degenerates $\Rightarrow$ slip line

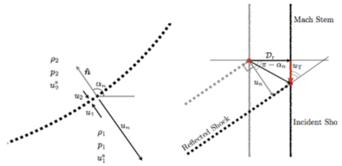
acoustic wave degenerates $\Rightarrow$ secondary shock

$$
\boxed{\frac{\partial^2 \alpha}{\partial t^2} - \overline{a}_N^2 \frac{\partial^2 \alpha}{\partial y^2} + \overline{\mathcal{D}} \frac{\partial}{\partial t} \left(\frac{\partial \alpha}{\partial y} \right)^2 = 0}
$$

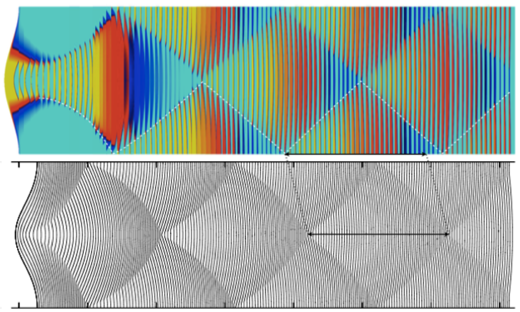
Numerical solution with an ondulated initial condition



Comparison with DNS



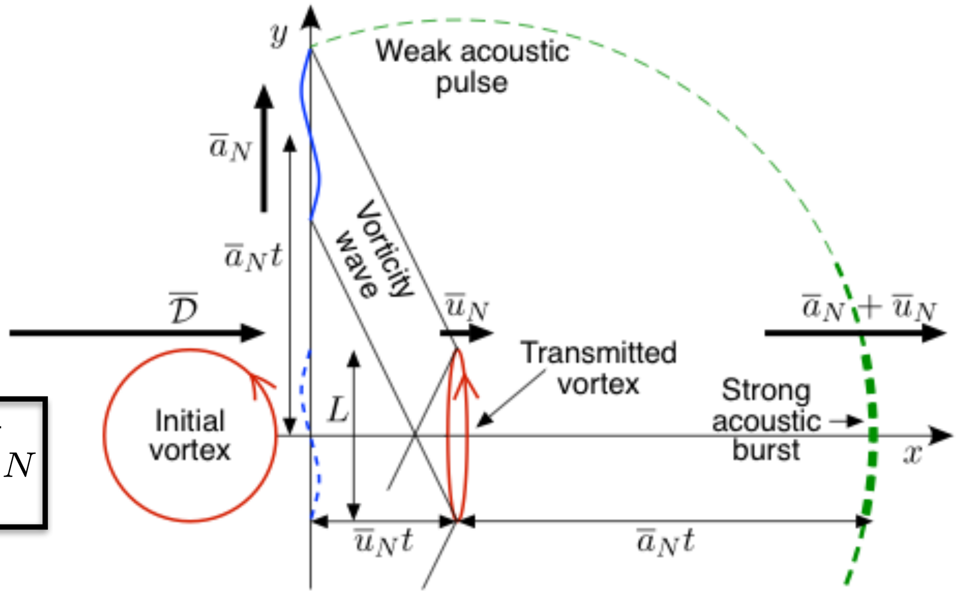
DNS



Model equation

Interaction between a strong shock and a weak vortex

$$\delta u_{1f}/a_u \ll \epsilon \equiv \overline{M}_N$$



short time scale

$$\dot{\alpha}_t \approx 2(\overline{a}_N/\mathcal{D})\delta u_{1f}$$

long time scale

$$\frac{\partial^2 \alpha}{\partial t^2} - \overline{a}_N^2 \frac{\partial^2 \alpha}{\partial y^2} + \mathcal{D}\frac{\partial}{\partial t}\left(\frac{\partial \alpha}{\partial y}\right)^2 = 0$$

Composite equation for the geometry of the front

$$\frac{\partial^2 \alpha}{\partial t^2} - \overline{a}_N^2 \frac{\partial^2 \alpha}{\partial y^2} + \mathcal{D}\frac{\partial}{\partial t}\left(\frac{\partial \alpha}{\partial y}\right)^2 = 2\frac{\overline{a}_N}{\mathcal{D}}\frac{\partial \delta u_{1f}}{\partial t}$$

$$\delta u_{1f}(y,t) \equiv u_e|_{x=-\mathcal{D}t}$$

Short living forcing term

Extension to 3-D: Shock turbulence interaction

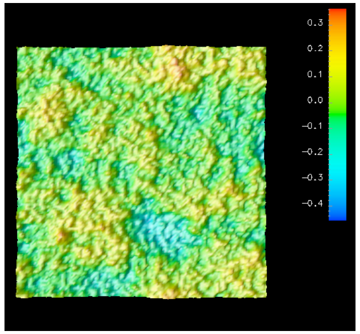
$$x = \alpha(y, z, t) \qquad \mathbf{u}_e(x, y, z, t) = (u_e, v_e, w_e) \qquad |\mathbf{u}_e|/a_u < \overline{a}_N/\mathcal{D} \ll 1$$

$$\overline{a}_N/a_u = O(1)$$

$$\frac{\partial^2 \alpha}{\partial t^2} - \overline{a}_N^2 \triangle \alpha + \mathcal{D}\frac{\partial}{\partial t}\left|\nabla \alpha\right|^2 = 2\frac{\overline{a}_N}{\mathcal{D}}\frac{\mathrm{d}}{\mathrm{d}t} u_e(-\mathcal{D}t, y, z, t)$$

Turbulence length-scale at the shock

L

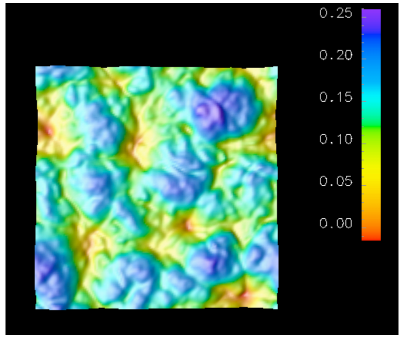


$t = 0^+$

Kolmogorov scale

$\lambda_K/L = 3 \times 10^{-2}$

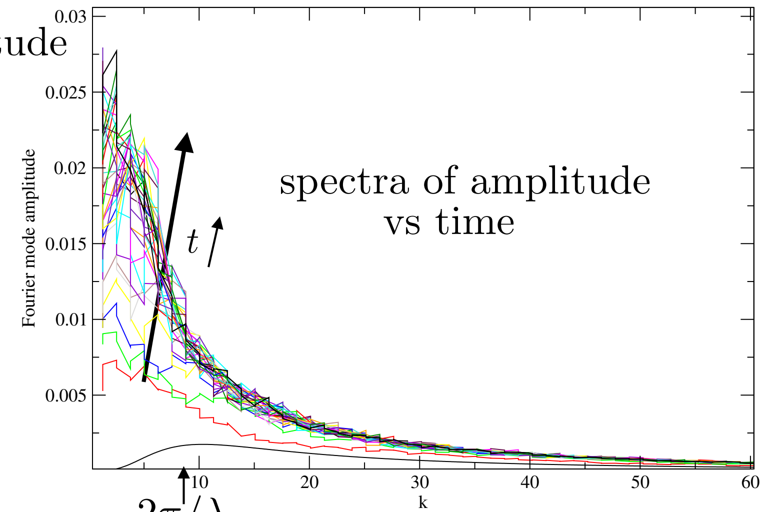
Shock surface after a while



$t = 4(\lambda_K/\overline{a}_N)$

The size and the amplitude of the cells increase with the time

amplitude



$2\pi/\lambda_K$

wave number

Nonlinear dynamics of cellular detonations

P. Clavin and R. Daou, *J. Fluid. Mech.* (2002)

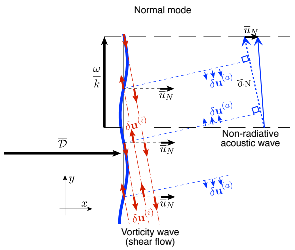
P. Clavin and B. Denet, *Phys. Rev. Lett.* (2002)

P. Clavin and F.A. Williams, *Phil. Trans. R. Soc.* (2012)

P. Clavin and G. Searby, *Cambridge Univ. Press* (2016)

P. Clavin, *Combust. Sci. Technol.* (2017)

Nonlinear theory for the cellular structure



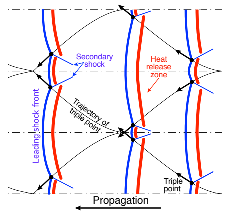
entropy wave
acoustic waves

direct influence

shock wave *heat release*

feedback loop

acoustic waves



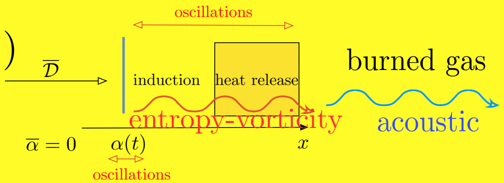
Weakly nonlinear analysis at the instability threshold

transverse instability $>$ longitudinal instability $\Rightarrow$ small heat release

Analytical solutions have been obtained in the Newtonian approximation for $\boxed{\text{two limiting cases}}$ (opposite conditions) :

Strongly overdriven detonation $\epsilon \equiv 1/M_u \approx M_N \ll 1, \quad (\gamma - 1) = O(\epsilon^2)$

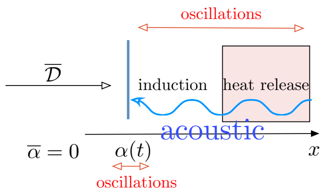
quasi-isobaric flow $+ \dfrac{q_m}{c_p \overline{T}_N} = O(\epsilon^2)$



dominant mechanisms : entropy-vorticity wave + radiating acoustic wave damped downstream in the burned gas flow

Near CJ conditions $\varepsilon^2 \equiv \dfrac{q_m}{c_p T_u} \ll 1, \quad (\gamma - 1) = O(\varepsilon) \quad (M_u - 1) = O(\varepsilon)$

transonic flow



dominant mechanism: upstream running acoustic wave through the detonation structure

The formation of triple point is similar in both cases

Weakly nonlinear analysis of the cellular patterns of strongly overdriven detonations near the stability limit in the Newtonian limit

$$M_N \ll 1 \qquad\qquad q_N \equiv \frac{q_m}{c_p \overline{T}_N} \ll 1$$

- Poincaré-Adronov (Hopf) bifurcation at small heat release $\quad q_N = O(M_N^2)$

Non zero and finite wavelength and frequency: $\quad \Lambda \approx d_N/M_N \qquad \omega \approx 2\pi/t_N$

$$d_N \equiv \overline{u}_N \tau_N \qquad t_N \equiv \tau_{reac}(T_N)$$

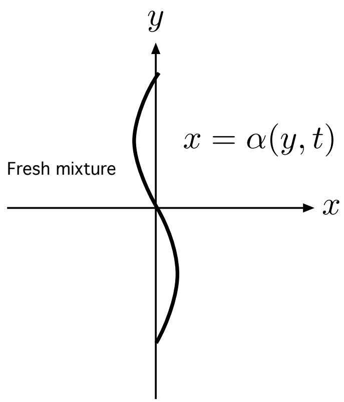
- Result: nonlinear equation for the detonation front

Non-dimensional form ($\alpha(y,t)$ scaled with d_N, y scaled with d_N/M_N, t scaled with t_N)

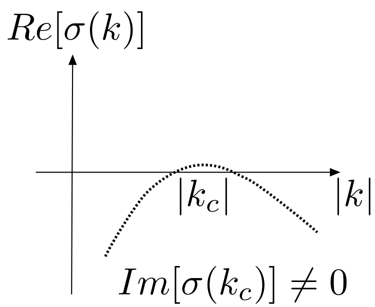
NL term $\qquad$ linear terms due to combustion

$$\frac{\partial^2 \alpha}{\partial t^2} - \frac{\partial^2 \alpha}{\partial y^2} + \frac{\partial}{\partial t}\left(\frac{\partial \alpha}{\partial y}\right)^2 = \left[q_N G(\alpha) - M_N \sqrt{q_N} D(\alpha)\right]$$

linear growth (quasi-isobaric mechanism) $\qquad$ linear damping (compressible effect)



$$\alpha(y,t) = \tilde{\alpha} e^{iky + \sigma t}$$



$$M_N \ll 1 \qquad q_N = O(M_N^2)$$

nonlinear dynamics of the lead shock

linear terms due to combustion

small linear growth rate

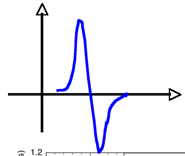
$$\frac{\partial^2\alpha}{\partial t^2} - \frac{\partial^2\alpha}{\partial y^2} + \frac{\partial}{\partial t}\left(\frac{\partial\alpha}{\partial y}\right)^2 = [q_N G(\alpha) - M_N\sqrt{q_N}D(\alpha)]$$

NL term

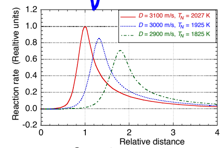
linear growth (quasi-isobaric mechanism)

linear damping (compressible effect)

$\Omega'_N(\mathrm{x})$



$\Omega(\Theta_N, \mathrm{x})$



Quasi-iobaric combustion instability mechanism

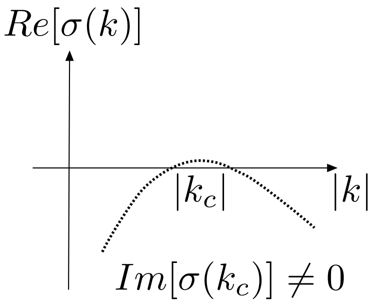
thermal sensitivity of the distribution of heat release rate

$$G_p(\alpha) = \int_0^\infty \Omega'_N(\mathrm{x})\alpha(\mathrm{t}-\mathrm{x},\mathrm{y})\mathrm{dx} \qquad \Omega'_N(\mathrm{x}) \equiv \partial\Omega/\partial\Theta_N$$

$$G(\alpha) = \beta_N(\gamma-1)\frac{\partial^2}{\partial t^2}G_p(\alpha) + \frac{\partial^2}{\partial y^2}G_w(\alpha) \qquad G^{(}_w\alpha) = \int_0^\infty \Psi(\mathrm{x})\alpha(\mathrm{t}-\mathrm{x},\mathrm{y})/\mathrm{dx} \qquad \Psi(\mathrm{x}) \equiv \Omega(\mathrm{x}) + \mathrm{d}(\mathrm{x}\Omega)/\mathrm{dx}$$

1-D galloping mechanism

modification due to wrinkling



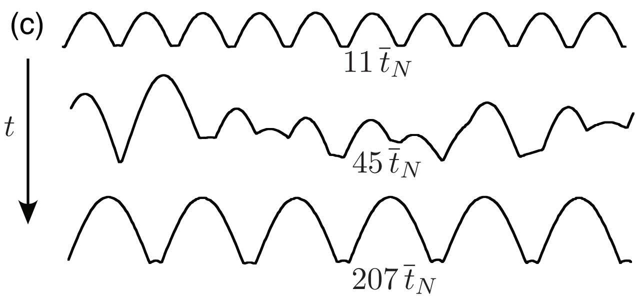
Damping term due to acoustic

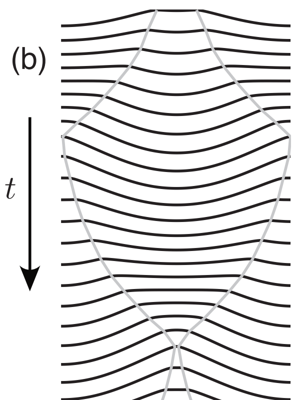
$D(\alpha) \approx -|k|\tilde{\alpha}$ in Fourier representation $\alpha(y,t) = \tilde{\alpha}(t)e^{iky}$

Numerical solution of the model equation

Long time selection

Good qualitative agreement with the experimental observation





selected wavelength $>$ most linearly unstable wavelength

Extensions

3-D geometry

$$\mathrm{x} = A(\mathrm{y},\mathrm{z},\mathrm{t}) \qquad \nabla^2 \equiv \frac{\partial^2}{\partial \mathrm{y}^2} + \frac{\partial^2}{\partial \mathrm{z}^2}$$

linear instability acoustic stabilisation

$$\boxed{\frac{\partial^2 A}{\partial \mathrm{t}^2}} - c^2\nabla^2 A + \frac{\partial |\nabla A|^2}{\partial \mathrm{t}} = \boxed{q_N L^{(i)}(A)} - 2\overline{M}_N\sqrt{q_N}\frac{\partial}{\partial \mathrm{t}}L^{(a)}(A) \qquad \overline{M}_N \ll 1 \qquad q_N = O(\overline{M}_N^2)$$

$$L^{(i)}(\alpha) = \boxed{\beta_N(\gamma-1)l^{(i)}_{\beta_N}(A)} + l^{(i)}_q(A) \left\{ \begin{array}{ll} l^{(i)}_{\beta_N}(A) = \dfrac{\partial^2}{\partial \mathrm{t}^2}\displaystyle\int_0^\infty \Omega'_N(\mathrm{x})A(\mathrm{y},\mathrm{z},\mathrm{t}-\mathrm{x})d\mathrm{x}, & \text{1-D instability} \\ l^{(i)}_q(\alpha) = \nabla^2 \displaystyle\int_0^\infty \Psi(\mathrm{x})A(\mathrm{y},\mathrm{z},\mathrm{t}-\mathrm{x})/d\mathrm{x} & \text{3-D instability} \end{array} \right.$$

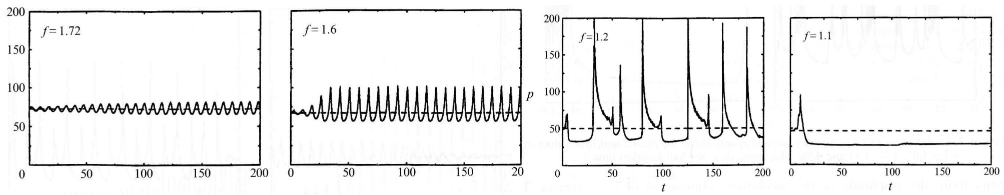
Secondary bifurcation: crossing the threshold of the planar instability $\beta_N(\gamma-1)q_N \nearrow$

$$\beta_N(\gamma-1)l^{(i)}_{\beta_N}(A) \rightarrow \boxed{-\frac{\partial}{\partial \mathrm{t}}\int_0^\infty \left[\Omega(\Theta_N(\mathrm{t}-\mathrm{x}),\mathrm{x}) - \Omega(\overline{\Theta}_N,\mathrm{x})\right]d\mathrm{x}} \quad \text{where} \quad \Theta_N(\mathrm{t}) - \overline{\Theta}_N = -\beta_N(\gamma-1)\frac{\partial A}{\partial \mathrm{t}}$$

linear $\longrightarrow$ nonlinear Rankine-Hugoniot

1-D instability

$$\boxed{\frac{\partial A}{\partial \mathrm{t}} = -q_N\int_0^\infty \left[\Omega(\Theta_N(\mathrm{t}-\mathrm{x}),\mathrm{x}) - \Omega(\overline{\Theta}_N,\mathrm{x})\right]d\mathrm{x}}$$

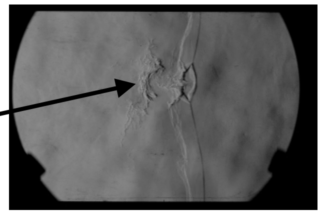


dynamical quenching

2-D instability

dynamical quenching $\Rightarrow$ pockets of fresh mixture

intermittency

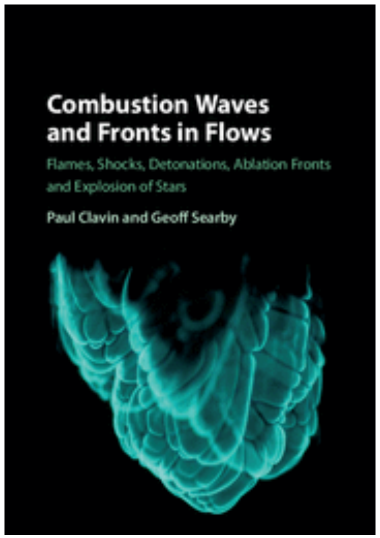


Conclusion

Most of the complex phenomena of the cellular
detonations in gases have been
deciphered by nonlinear analytical studies

Amongst the phenomena that are still not fully understood
is the deflagration to detonation transition, a problem of importance for safety reason
and also for the explosion of supernovae I

Another open problem is the fate of the strong shock formed during the gravitational
collapse of the iron core of a massive stars at the end of their life (supernovae II)



Cambridge University Press 2016